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From Discrete Causal Sets to a Spacetime Continuum

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From discrete causal sets
to a spacetime continuum

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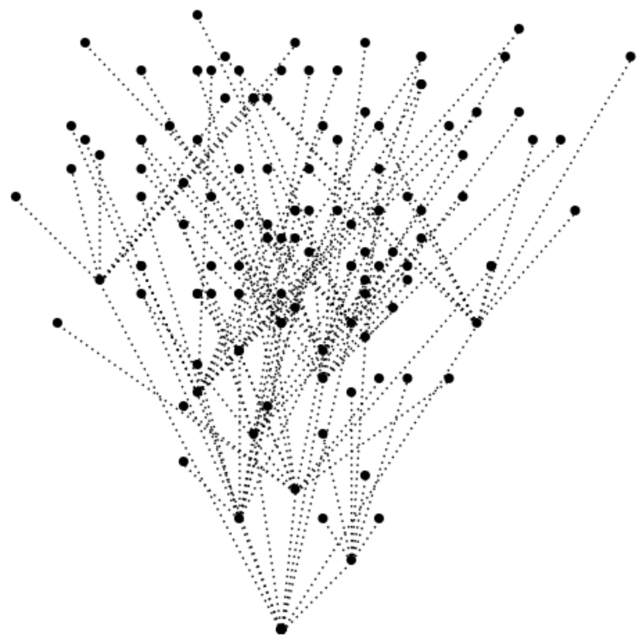
Causal Sets

What do you keep when you make spacetime discrete?

One choice: (relativistic) time, i.e., causal relations $p \prec q$



Causal set:

– partial order $\Rightarrow$ causal order

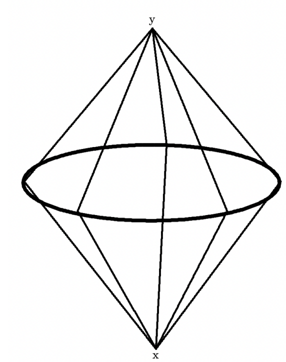
– no closed timelike curves: no x, y for which $x \prec y$ and $y \prec x$

– discrete: for any x, y, $\{z|x \prec z \prec y\}$ is finite

Basic ingredient: causal diamond/Alexandrov interval/order interval:

For x, y with $x \prec y$,

$$I(x, y) = \{z|x \prec z \prec y\}$$



Why?

Hawking, King, McCarthy; Malament:

For a spacetime, causal structure + volume element determines geometry

Causal sets are simplest discrete embodiment

– causal structure: built in

– volume element: number of points

"Order + Number = Geometry"

But how well do causal sets approximate the continuum?

Two questions:

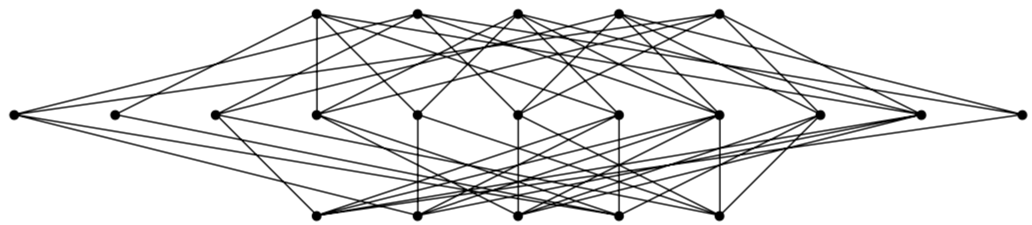
- start with spacetime manifold, approximate by causal set
- start with causal set, find suitable “smoothed” spacetime

First direction is in good shape:

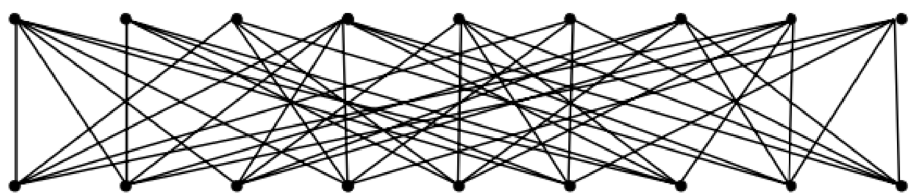
- “Poisson sprinkling” of points approximates manifold
- Can reconstruct topology, volume, curvature, d’Alembertian, etc.
- Open questions about defining “close” sets
- Locality can be tricky

But. . .

most causal sets are nothing at all like manifolds



KR order



2-layer set

- Almost all causal sets are Kleitman-Rothschild orders

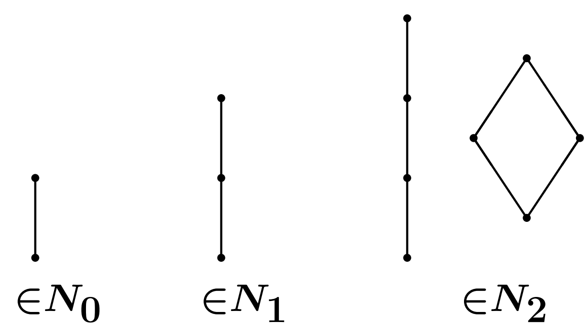
$$\frac{\text{\# of KR orders with } n \text{ elements}}{\text{\# of causal sets with } n \text{ elements}} = 1 + \mathcal{O}\left(\frac{1}{n}\right)$$

- Almost all remaining causal sets are two-layer sets
- Then four-layer, five-layer, . . .
- Manifoldlike causal sets are of measure zero

Causal set path integral

How to make invariants from a causal set: count causal diamonds

Invariant $N_J(C)$: number of (open) intervals in set C with exactly J points



For continuum spacetime, causal diamond volumes depend on curvature

Benincasa-Dowker action:

$$\frac{1}{\hbar} I_{BD}(C) = \left(\frac{\ell}{\ell_p}\right)^2 (n - N_0 + 9N_1 - 16N_2 + 8N_3)$$

For manifold-like causal set, I_{BD} approximates Einstein-Hilbert action

Choose class Ω of causal sets

Path sum:

$$\mathcal{Z}(\Omega) = \sum_{C \in \Omega} \exp\left\{\frac{i}{\hbar} I_{BD}(C)\right\}$$

- Result 1 (S. Carlip and S. P. Loomis):

  For 2-layer sets, $\mathcal{Z} \sim 2^{-cn^2}$ for a large range of coupling constants

Sketch of proof:

- for two layers, only $N_0 \neq 0$, so $I_{BD} \sim (n - N_0)$
- maximum number of links is $N_{max} = \frac{n^2}{4}$
- write $N_0 = pN_{max}$

$$\Rightarrow \; \mathcal{Z} \sim \sum_p \mu_n(p)\, e^{-i\beta p n^2}$$

- use combinatorial arguments to bound measure $\mu_n(p)$
- approximate sum as integral, use steepest descent (carefully!)

- Result 2 (A. Mathur, A. A. Singh, and S. Surya):
  - For a very large class of layered causal sets, same suppression *but* with "link action": $\boldsymbol{I}_{\textit{link}} \sim (\boldsymbol{n} - \boldsymbol{N_0})$

Reminder: $\boldsymbol{I}_{\textit{BD}} \sim \boldsymbol{n} - \boldsymbol{N_0} + \boldsymbol{9N_1} - \boldsymbol{16N_2} + \boldsymbol{8N_3}$

$$\boldsymbol{I}_{\textit{link}} \sim \boldsymbol{n} - \boldsymbol{N_0} + \cancel{\boldsymbol{9N_1}} - \cancel{\boldsymbol{16N_2}} + \cancel{\boldsymbol{8N_3}}$$

Proof: same as before, but more complicated combinatorics for $\boldsymbol{\mu_n}(\boldsymbol{p})$

- Result 3 (P. Carlip, S. Carlip, and S. Surya):
  For KR orders, $\boldsymbol{I}_{\mathit{link}}$ is almost always equal to $\boldsymbol{I}_{\mathit{BD}}$

Basic argument:

$\in \boldsymbol{N_1}$ (thin) $\in \boldsymbol{N_6}$ (fat)

For large KR order, “thin” causal diamonds are rare
$\Rightarrow \boldsymbol{N_J \sim n}$ is subdominant in action

- Result 4 (P. Carlip, S. Carlip, and S. Surya, in progress):
  Same is almost certainly true for almost all layered causal sets

Path integral suppresses
– a very large class of “bad” causal sets
– but *not* manifoldlike causal sets!

Some remaining problems:

- There are almost certainly other “bad” causal sets
  Can they be classified, and are they suppressed?
- How does one give a general characterization of “manifold-like” sets?
- B-D action was derived from manifold Einstein-Hilbert action
  Can it be obtained from first principles?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