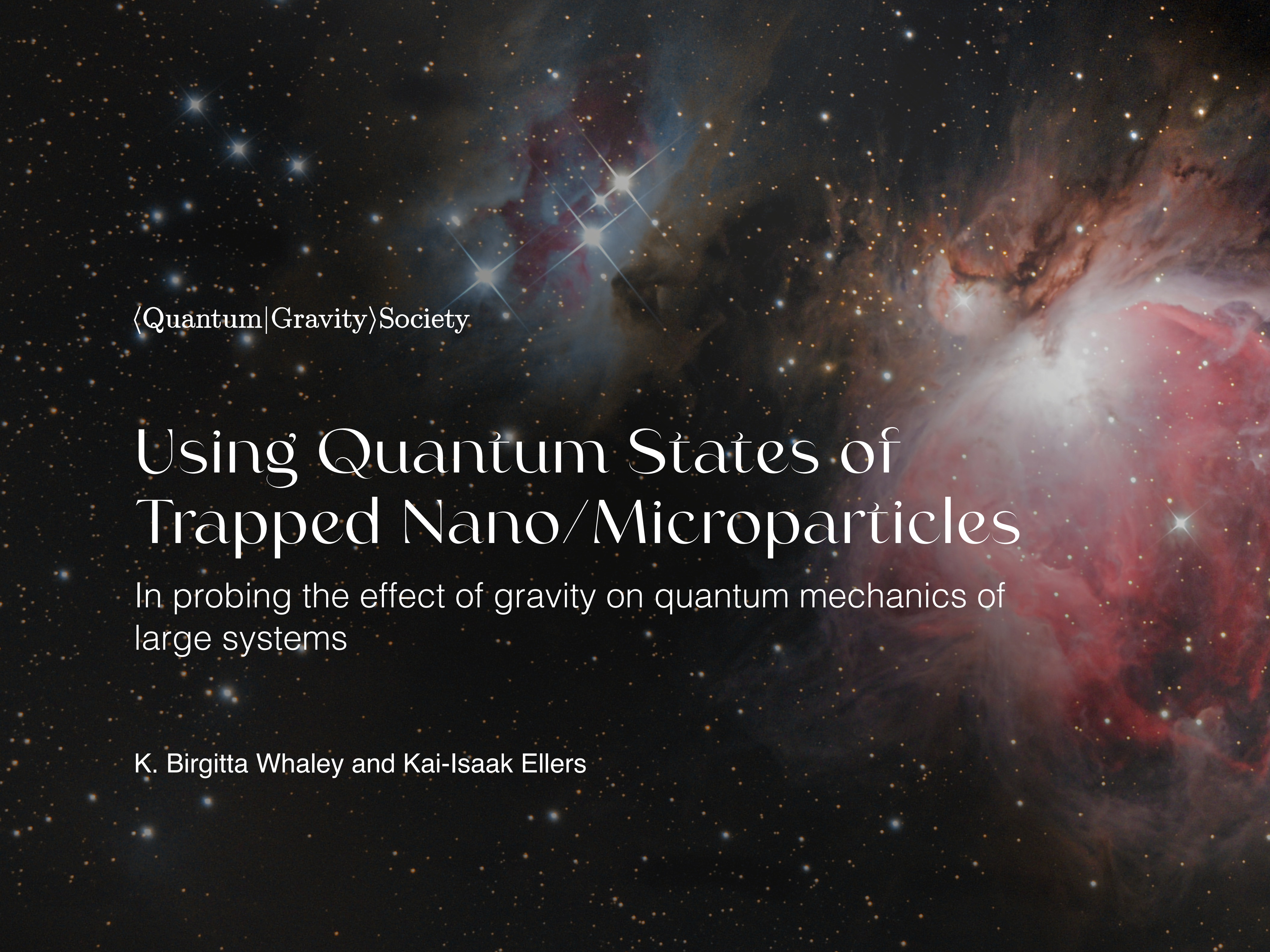


⟨Quantum|Gravity⟩Society

# Using Quantum States of Trapped Nano/Microparticles

## In probing the effect of gravity on quantum mechanics of large systems

K. Birgitta Whaley and Kai-Isaak Ellers

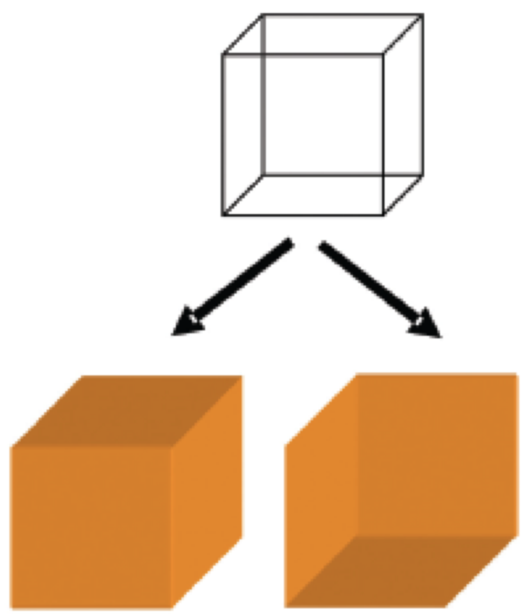
# Using quantum states of trapped nano/micro-particles to probe the effect of gravity on quantum mechanics of large systems

Kai-Isaak Ellers and **K. Birgitta Whaley**
University of California, Berkeley

Jordan Wilson-Gerow
California Institute of Technology

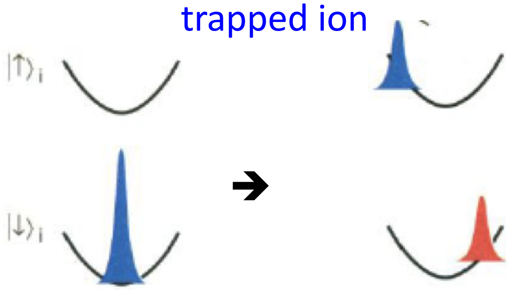
# Quantum superpositions:

## visual representation with ambiguous cube



F.A. Wolf, *Taking the Quantum Leap: The New Physics for Nonscientists*, New York: Harper & Row (1989).

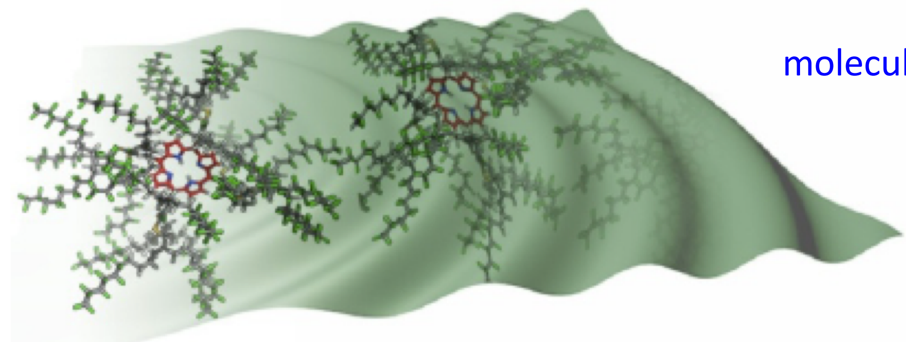
trapped ion




1 atom, m~$10^{-26}$ kg, Δx~80 nm

Monroe et al. Science 272, 1131 (1996)

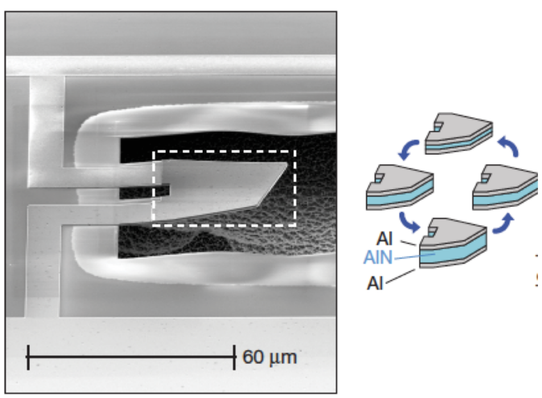
molecular diffraction



2000 atoms, m~$10^{-23}$ kg, Δx>500 nm

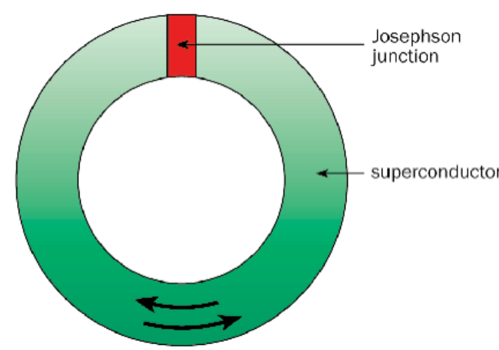
Fein et al. Nat. Physics 15, 1242 (2019)

mechanical resonator




$10^{13}$ atoms, m~$10^{-13}$ kg, Δx~2x$10^{-16}$ nm

McConnell et al. Nature 464, 697 (2010)




**flux superposition**

$$\Psi = |\circlearrowleft\rangle + |\circlearrowright\rangle$$

$10^{13}$ electrons, L~560 $\mu$m, $\Delta I_p$=2-3 A

but only $\sim 10^4$
$e^-$ in different modes...

Friedman et al. Nature 406, 43 (2000)

Korsbakken et al.
Phys.Scripta T137, 014022 (2009)

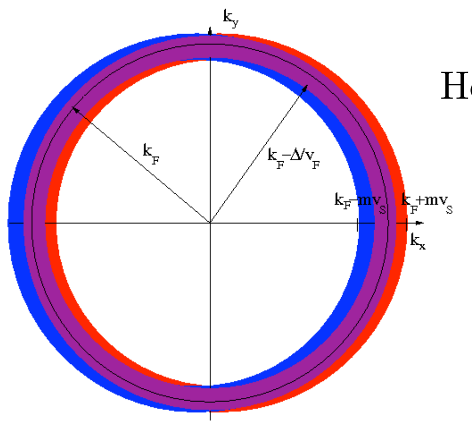
# Flux superposition size: $\Psi = |A\rangle + |B\rangle$

difference in magnetic moment ("extensive difference')

$\Delta\mu = A\delta I_p \sim 10^6 - 10^{10}$ $\delta I_p$ = difference in persistent current, clockwise/anticlockwise

Leggett J. Phys. Cond. Matt.14**,** R415 (2002)

- Measurement-based measure, i.e., operational measure
- Minimum number of particles that have to be measured to distinguish branches



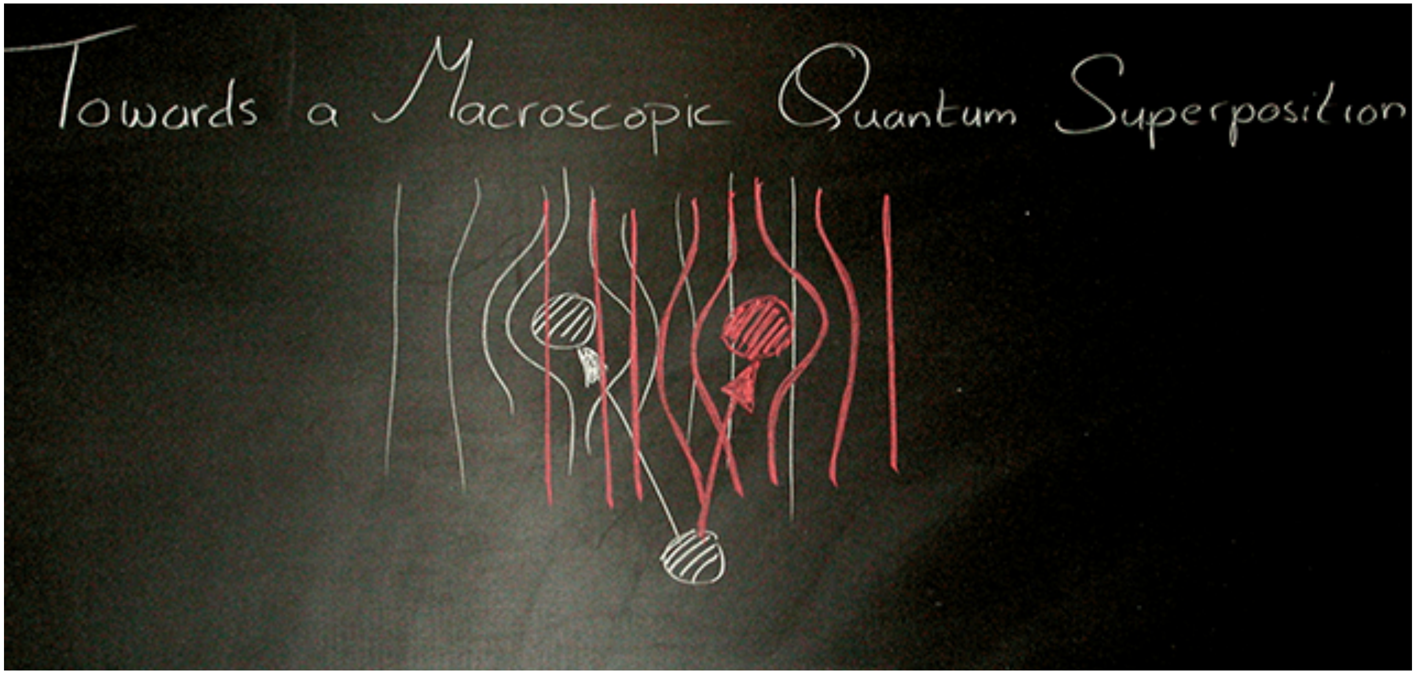

How many electrons are in different modes $\mathbf{k},\sigma$ in the two branches $|\circlearrowleft\rangle$ and $|\circlearrowright\rangle$?

$$\Delta N_{tot} = \sum_{\mathbf{k},\sigma} \langle\circlearrowleft|\hat{c}^\dagger_{\mathbf{k},\sigma}\hat{c}_{\mathbf{k},\sigma}|\circlearrowleft\rangle - \langle\circlearrowright|\hat{c}^\dagger_{\mathbf{k},\sigma}\hat{c}_{\mathbf{k},\sigma}|\circlearrowright\rangle$$

$\Delta N_{tot} \sim 10^2 - 10^4$

similar if measure Cooper pairs

Korsbakken et al. PRA 74, 042106 (2007)
Korsbakken et al. Physica Scripta **T137**, 014022 (2009)
Korsbakken et al. Europhys. Lett. **89**, 30003 (2010)



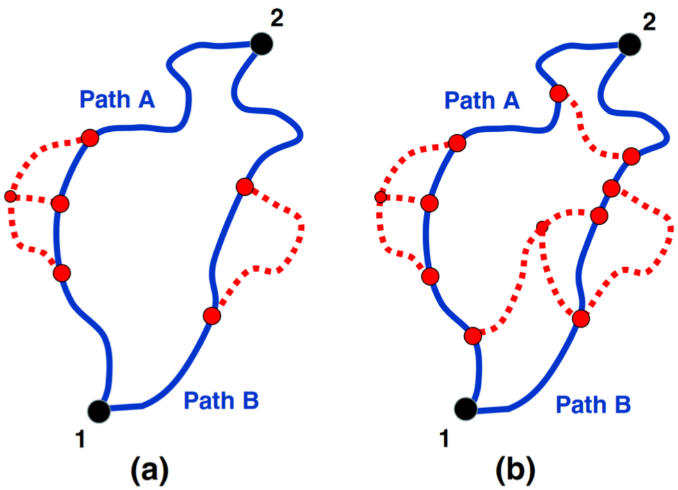

D. Bouwmeester

- **How does gravity affect a mass superposition?**
- Can macroscopic superpositions tell us whether gravity is quantized?
- Does gravity destroy large quantum superpositions to ‘collapse’ the state ?
- Can gravity create entanglement?

# CWL Propagators

N replicas

Two-path process in conventional QM/QFT vs CWL




QM/QFT

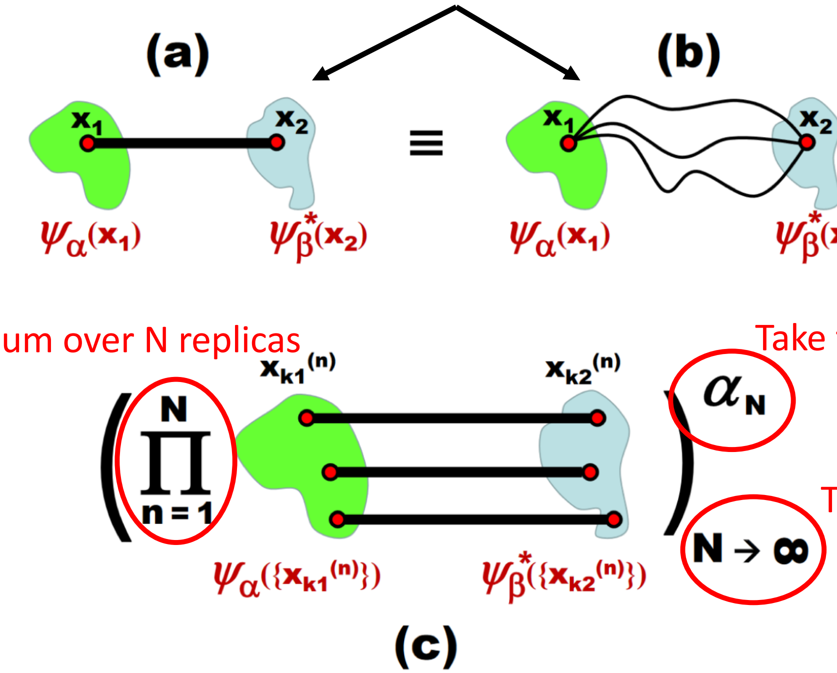
CWL

QM Propagator

In CWL, sum over N replicas

Take the $\alpha_N$ root

Take N to infinity

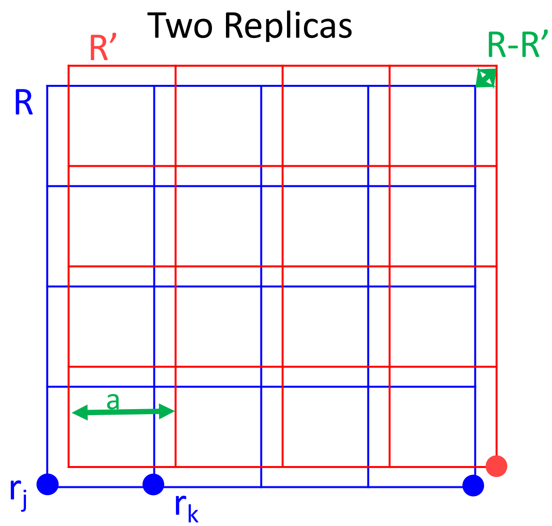



$$\mathcal{K}(\beta,\alpha) = \lim_{N\to\infty}\left[\int_\alpha^\beta \mathcal{D}q_1 \ldots \int_\alpha^\beta \mathcal{D}q_N\, e^{iS[\{q_j\}]}\right]^{\frac{1}{N}}$$

[1] Wilson-Gerow and Stamp, Physical Review D, 2022

J. Wilson-Gerow, 2022

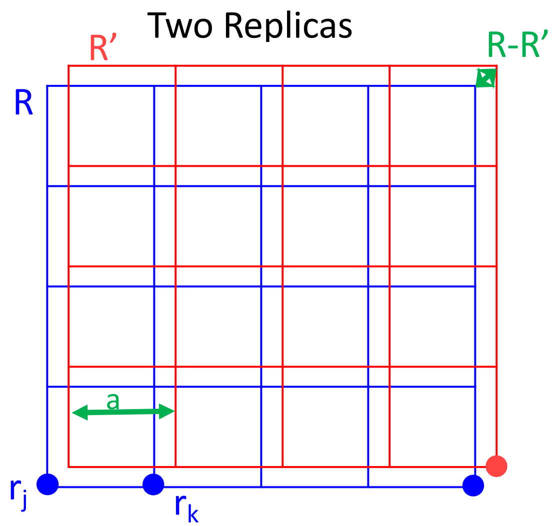
# CWL Interactions between replicas for finite extended body

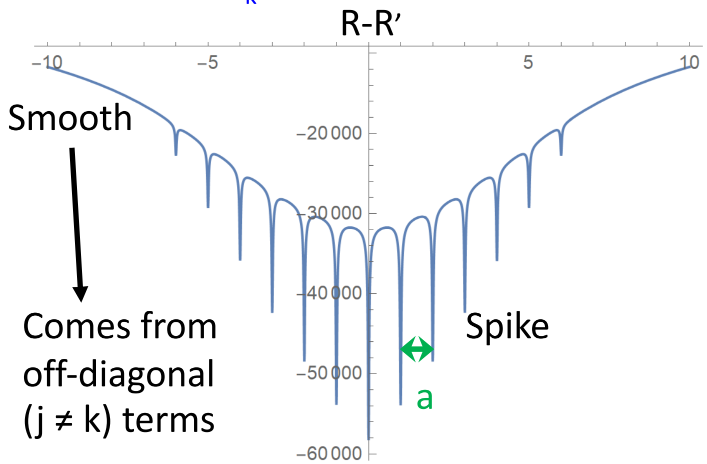



$N_{at}$ atoms in extended body

Consider effects in
non-relativistic system, weak gravitational field, Newtonian gravity

# CWL Interactions between replicas for finite extended body



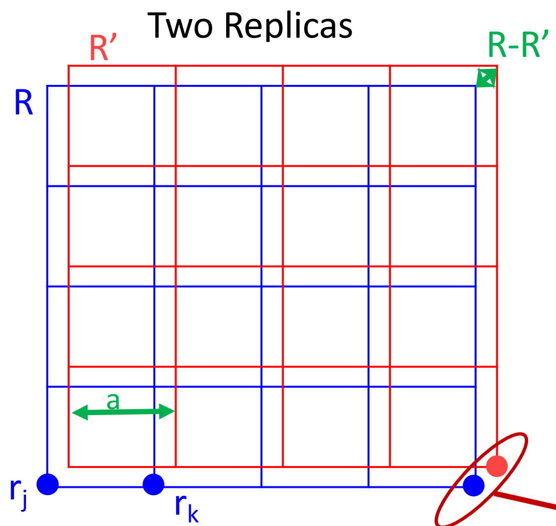



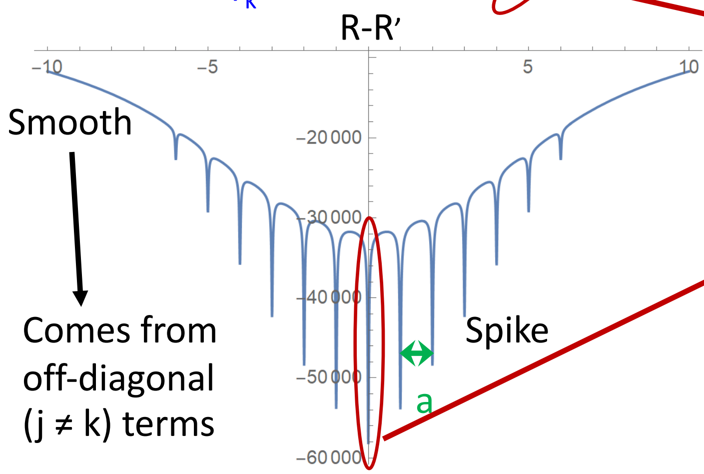

$N_{at}$ atoms in extended body

$$\tilde{S}_{CWL} = \int_{t1}^{t2} dt \sum_{j,k=1}^{N_{at}} G m_j m_k \frac{1}{|\vec{R}(t) - \vec{R}'(t) + \vec{r}_j - \vec{r}'_k|} \mathrm{Erf}\left( \frac{|\vec{R}'(t) - \vec{R}'(t) + \vec{r}_j - \vec{r}'_k|}{2\sigma} \right)$$

J. Wilson-Gerow, PhD Thesis UBC, 2021

# CWL Interactions between replicas for finite extended body

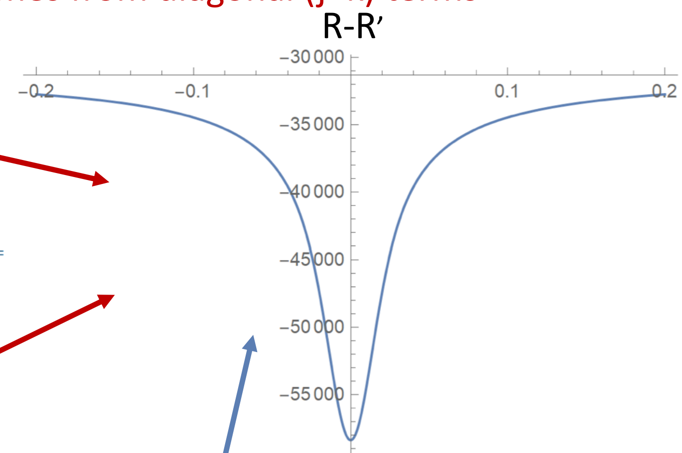



$N_{at}$ atoms in extended body

$$\tilde{S}_{CWL} = \int_{t1}^{t2} dt \sum_{j,k=1}^{N_{at}} G m_j m_k \frac{1}{|\vec{R}(t) - \vec{R}'(t) + \vec{r}_j - \vec{r}'_k|} \mathrm{Erf}\left(\frac{|\vec{R}'(t) - \vec{R}'(t) + \vec{r}_j - \vec{r}'_k|}{2\sigma}\right)$$

Spike comes from diagonal (j=k) terms



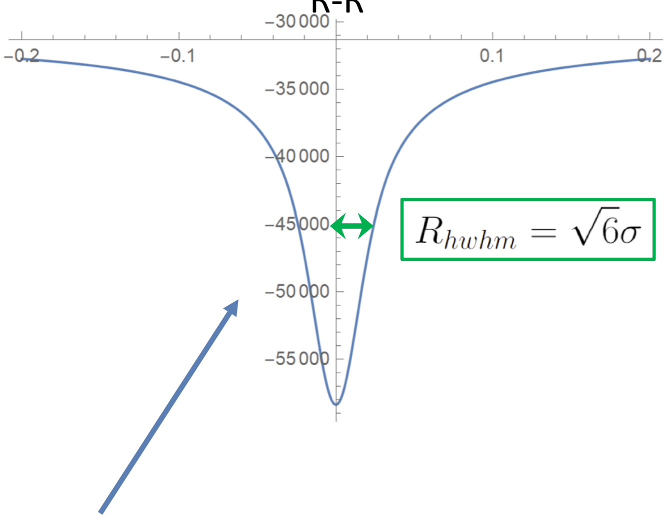

J. Wilson-Gerow, PhD Thesis UBC, 2021

$$V_{spike}(\vec{R}) = -\frac{GMm}{|\vec{R}|} \mathrm{Erf}\left(\frac{|\vec{R}|}{2\sigma}\right) = -\frac{GMm}{\sqrt{\pi}\sigma} + \frac{GMm}{12\sqrt{\pi}\sigma^3}|\vec{R}|^2 + \ldots$$

# Constraints on Nanoparticle Wavefunction

- Nanospheres in Aspelmeyer lab[2]: $\sigma_{expt} = \sqrt{\dfrac{\hbar}{M\omega_0}} = 1.5 \times 10^{-11}$
- Atomic RMS fluctuations at 15 K [3]: σ ≈ 3 pm

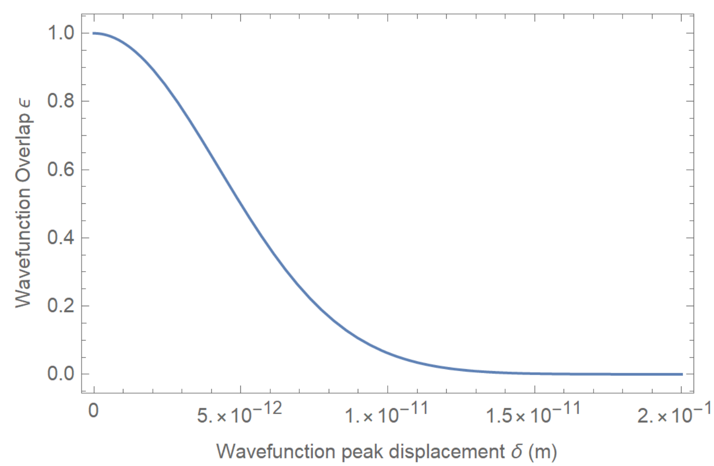
$SiO_2$, silica; diameter 150 nm




$$R_{hwhm} = \sqrt{6}\sigma$$

$$V_{spike}(\vec{R}) = -\frac{GMm}{|\vec{R}|}\,\mathrm{Erf}\left(\frac{|\vec{R}|}{2\sigma}\right) = -\frac{GMm}{\sqrt{\pi}\sigma} + \frac{GMm}{12\sqrt{\pi}\sigma^3}|\vec{R}|^2 + \ldots$$

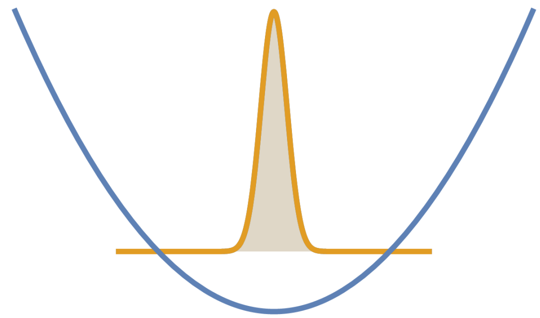
- Overlap between atomic wave functions falls off exponentially with separation:





[2] H Rudolph et al., arXiv, 2022
[3] RT Downs et al., American Minerologist, 1990

# no CWL effect for “simple” observables



- Assume the following

  1) initial state is Gaussian

  2) action is quadratic (i.e. harmonic oscillator potential)

  3) final state has coordinates of all replicas equal (e.g., position measurement)

- 1) & 2) $\Longrightarrow$ final state separates into replica COM and replica relative coordinates

$$X \equiv N^{-1} \sum_j x_j \qquad\qquad r_j \equiv x_j - X$$

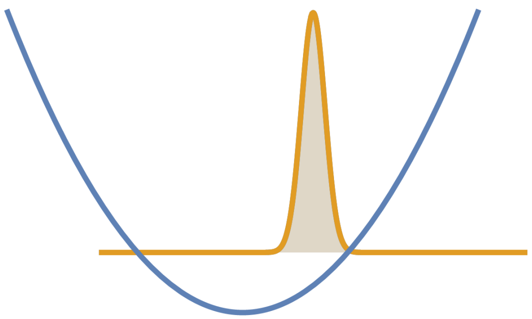
- 3) $\Longrightarrow$ final relative coordinates are all zero

- Since initial coordinates are integrated over, there is no dependence of final state on relative coordinates

- CWL action depends only on relative coordinates $\Longrightarrow$ no CWL effect on final measurement of position

# state projection of time-evolved displaced oscillator

- Consider displaced oscillator
- State to state propagator $\mathcal{K}(\beta, \alpha)$
- Use perturbation theory to evaluate this…



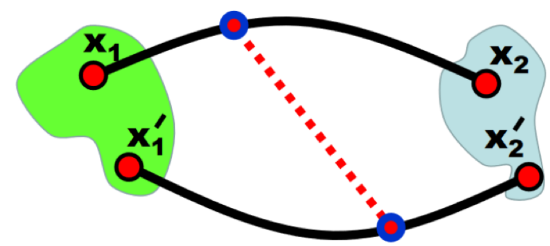
Harmonic oscillator frequency $\omega$

Spike potential frequency $\Omega_{spike} = \sqrt{\frac{Gm}{6\sqrt{\pi}\sigma^3}}$

Effective harmonic CWL frequency $\Omega^2 = \omega^2 + \frac{Gm}{6\sqrt{\pi}\sigma^3}$

# CWL Perturbation Theory

Lowest order CWL contribution in $l_p^2$ approximation [4]




- Expand in powers of $l_p^2 = \frac{\hbar G}{c^3}$

- Truncating at order $l_p^2$ → simplifies to two-replica interactions

- Calculate transition element between $|\alpha\rangle$ and $|\beta\rangle$: [4]

$$\mathcal{K}(\beta,\alpha) \sim K_0^{-1}(\beta,\alpha) \int_\alpha^\beta \mathcal{D}q \int_\alpha^\beta \mathcal{D}q' e^{i(S[q]+S[q'])/\hbar}(1+iS_{CWL}[q,q']/\hbar)$$

where

$$\int_\alpha^\beta \mathcal{D}q = \int dx_1 dx_2 \langle\beta|x_2\rangle \langle x_1|\alpha\rangle \int_{x_1}^{x_2} \mathcal{D}x$$

"Bare" transition element (no CWL)

CWL action from spike potential:

$$S_{CWL} = -\int_{t1}^{t2} V_{spike}(|\vec{R}-\vec{R}'|)dt \approx \int_{t1}^{t2} \left(\frac{GMm}{\sqrt{\pi}\sigma} - \frac{GMm}{12\sqrt{\pi}\sigma^3}|\vec{R}-\vec{R}'|^2\right)dt$$

- To avoid zeros of denominator, choose initial and final states such that $K_0(\beta,\alpha) = 1$


[4] Wilson-Gerow and Stamp, Physical Review D, 2022

# Displaced oscillator with CWL perturbation theory

- To avoid zeros of denominator, choose states such that $K_0(\beta, \alpha) = 1$ , e.g., project onto zeroth order time evolved state

$$\left\langle R^{(\prime)}|\alpha\right\rangle = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} \exp\left\{-\frac{M\omega}{2\hbar}\left(R^{(\prime)} - \frac{a}{\sqrt{2}}\right)^2\right\}$$

$$\left\langle R^{(\prime)}|\beta\right\rangle = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \exp\left\{\frac{M\omega}{2\hbar}\left[-R^{(\prime)2} - \sqrt{2}aR^{(\prime)}e^{-i\omega T} - \frac{a^2}{2}\cos\omega T e^{-i\omega T}\right]\right\}$$

Replicas R and R'

Displaced H.O. ground state

State chosen such that $K_0(\beta, \alpha) = 1$

Result: $$\mathcal{K}(\beta, \alpha) = e^{i\omega T/2}\sqrt{\frac{2\omega\Omega}{2\omega\Omega\cos\Omega T + i(\Omega^2 + \omega^2)\sin\Omega T}}$$

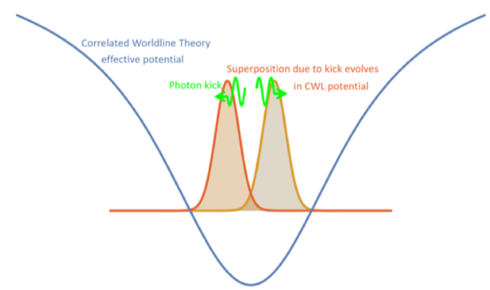
Consistency check: $G \to 0 \implies \mathcal{K}(\beta, \alpha) = 1$

This suggests a non-unitary CWL correction to quantum state dynamics, so now consider non-gaussian state:

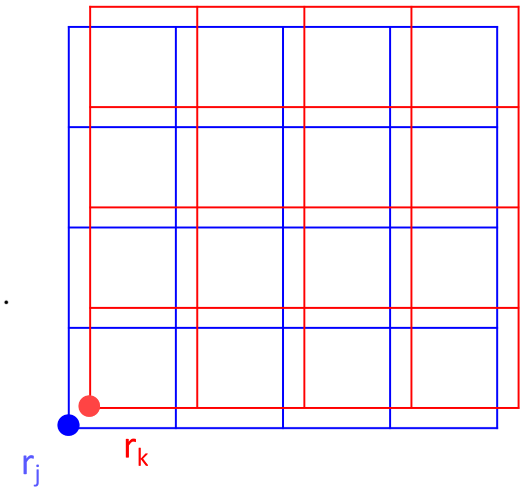
**but**

initial state is product state

perturbation theory is problematic…



# problem with perturbation theory…




Expansion parameter: $l_p^2 = \frac{\hbar G}{c^3}$

But in CWL, G → G/N to compensate for N replicas: $V(r) = \frac{Gm}{r} \rightarrow \frac{\frac{G}{N}Nm}{r} \rightarrow \frac{Gm}{r}.$

CWL interaction → $V_{CWL} = \frac{Gm^2}{2N}\sum_{j\neq k}\frac{1}{|r_j - r_k|}$, sum over different replicas

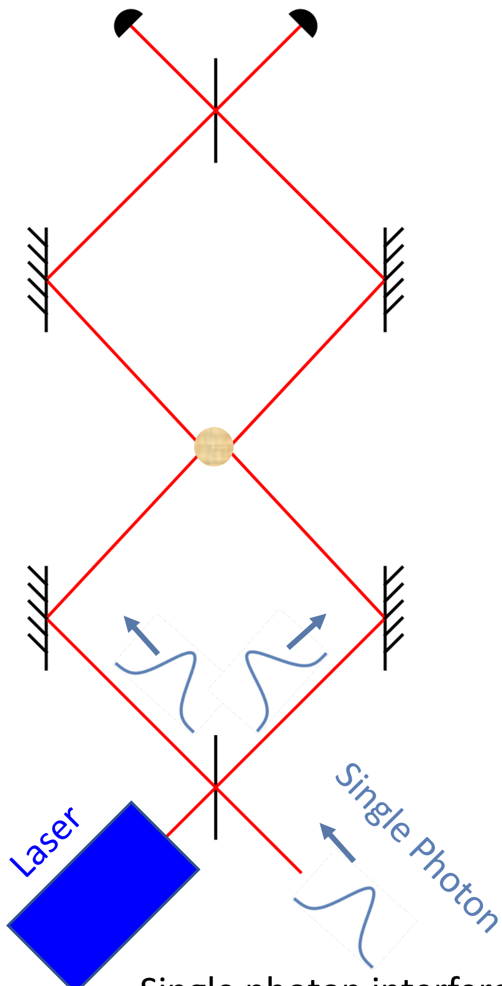
$V_{CWL}$ has $N^2$ terms ⟹ $V_{CWL} \propto \frac{G}{N}\sum_{j\neq k}\frac{1}{|r_j - r_k|} \propto \frac{G}{N}N^2 \propto GN$

Since N → ∞, cannot do perturbation expansion in GN

So, we should be analyzing quantum dynamics non-perturbatively, i.e., fully summing all Newtonian contributions

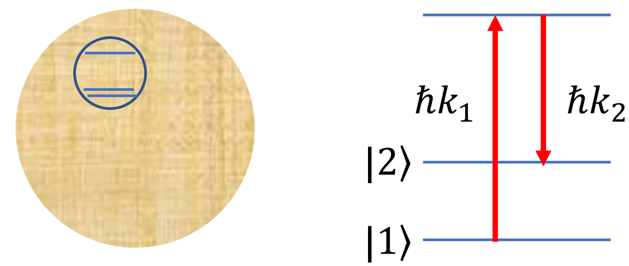
# Generating Momentum Superpositions of Nano/Microparticles

$\psi(x) \sim \psi_0(x)e^{+ipx} + \psi_0(x)e^{-ipx}$



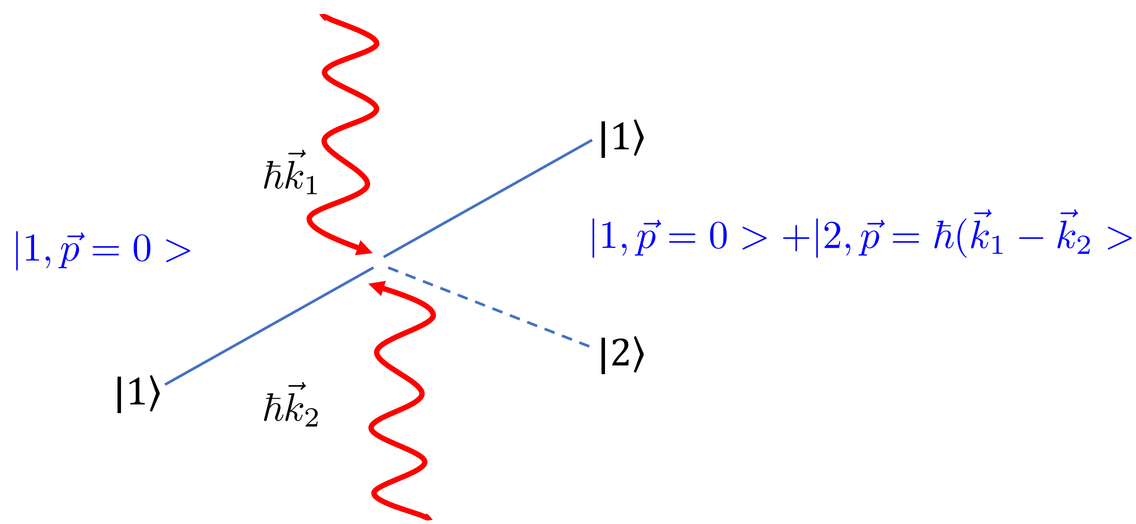

Single photon interference

Optical Raman on NV Center in diamond

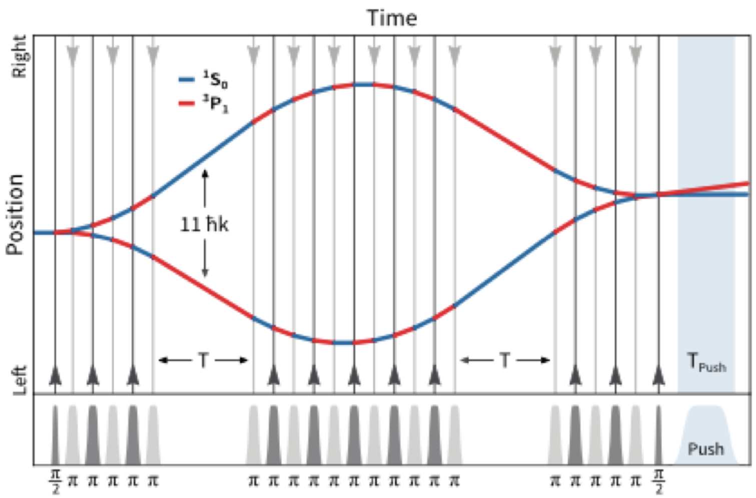



can extend to $\sim 8\hbar k$

H. Mueller



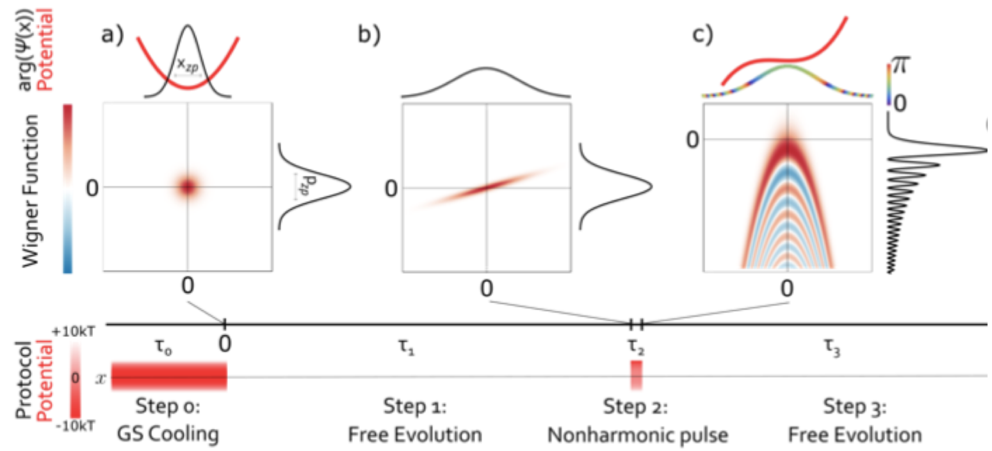
single photon interactions
can achieve large momentum transfer (Kasevich)




momentum transfer up to $141\,\hbar k$

Rudolph et al. (Hogan group) PRL 124, 083604 (2020)

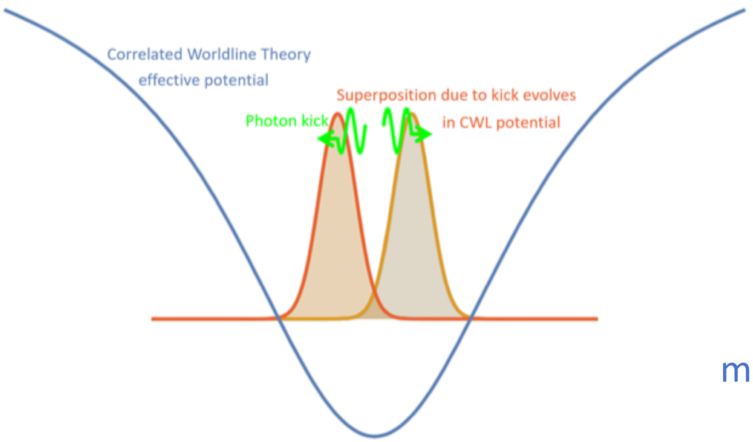
Pulsed cubic optical potential creates non-Gaussian states




$$\psi(x) = \exp(-\frac{1}{4\sigma^2}x^2 + ibx^3) + \exp(-\frac{1}{4\sigma^2}x^2 - ibx^3)$$

Neumeier et al. 2207.12539

# Dynamics of momentum superposition state in harmonic potential: I




position basis transition element at t:

$$p(y) = \lim_{N \to \infty} |\, \langle y|^{\otimes N}\, e^{-iH_{CWL}t} \hat{\mathcal{O}}_k \, |\Psi_0\rangle \,|^{2/N}$$

momentum 'kick' operator generating superposition

CWL correlated ground state

Probability distribution at time t:

$$P(y) = \frac{p(y)}{\int_{-\infty}^{\infty} p(z)dz}$$

renormalize on account of non-unitary nature of CWL evolution

# Dynamics of momentum superposition state in harmonic potential: II

$$\psi(x) = \exp(-\frac{1}{4\sigma^2}x^2 + ipx) + \exp(-\frac{1}{4\sigma^2}x^2 - ipx)$$

*N* replicas →

$$\Psi(\{x_j\}) \propto \sum_{\{\lambda_j\}=\{\pm 1\}} e^{ip\sum_{j=1}^{N}\lambda_j x_j}\Psi_0(\{x_j\})$$

↓ time evolution

non-perturbative

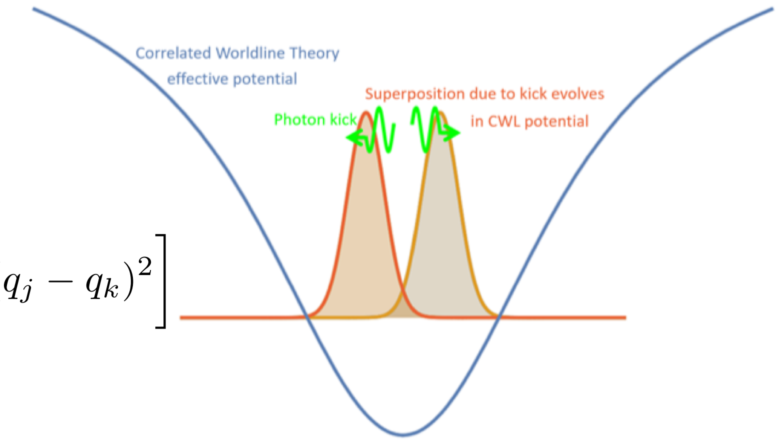
$$\Psi(\{y_j\}) = \sum_{\{\lambda_j\}=\{\pm 1\}} \int dx_1 ... dx_N \, e^{ip\sum_{j=1}^{N}\lambda_j x_j}\Psi_0(\{x_j\}) \int_{x_1}^{y_1} \mathcal{D}q_1 ... \int_{x_N}^{y_N} \mathcal{D}q_N \, e^{iS[\{q_j\}]}$$

$$p(y) = \lim_{N\to\infty} |\Psi(\{y_j\} = y)|^{2/N} \longrightarrow P(y) = \frac{p(y)}{\int_{-\infty}^{\infty} p(z)dz}$$

probability distribution at time t

# Dynamics of momentum superposition state in harmonic potential: III

evaluate $\Psi(\{x_j\})$ for CWL action:




$$S[\{q_j\}] = \int_0^T dt \left[ \sum_{j=1}^{N} \left( \frac{m}{2}\dot{q}_j^2 - \frac{m\omega^2}{2} q_j^2 \right) - \frac{m\Omega_{spike}^2}{4N} \sum_{j,k} (q_j - q_k)^2 \right]$$

$$\Omega_{spike} = \sqrt{\frac{Gm}{6\sqrt{\pi}\sigma^3}}$$

expand to order

$$\epsilon \equiv \frac{\Omega_{spike}^2}{\omega^2}$$

$$\Psi(y,t) = \exp\left(\frac{i(h-g)N\Theta^2}{2}\right) \sum_{n=0}^{N} \binom{N}{n} \exp\left[ i(h-g)p\Theta(2n-N) + \frac{ip^2}{2N} h(2n-N)^2 \right] \qquad \Theta = \frac{ym\omega}{\sin\omega t}$$

$\dashrightarrow P(y)$

# Summary and next steps

- Momentum superposition states of trapped nano/microparticles ($r_M \sim 10^{-7}$ m) show promise for analyzing the effects of gravity on dynamics of non-classical quantum states

- Microparticles ($r_M \gtrsim 10^{-6}$ m) have larger CWL effects, since can ensure that particle sits inside spike potential but larger momentum kick then needed to generate superposition…
  see Jordan Wilson-Gerow talk Thursday morning

- More detailed analysis of generation and control of superposition states

- Influence of environmental decoherence on massive superpositions

- Non-perturbative (in the sense of full summation of Newtonian contributions) CWL calculations are feasible for trapped massive particles

# Acknowledgements

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(G5)

CIQC Challenge Institute for Quantum Computation

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