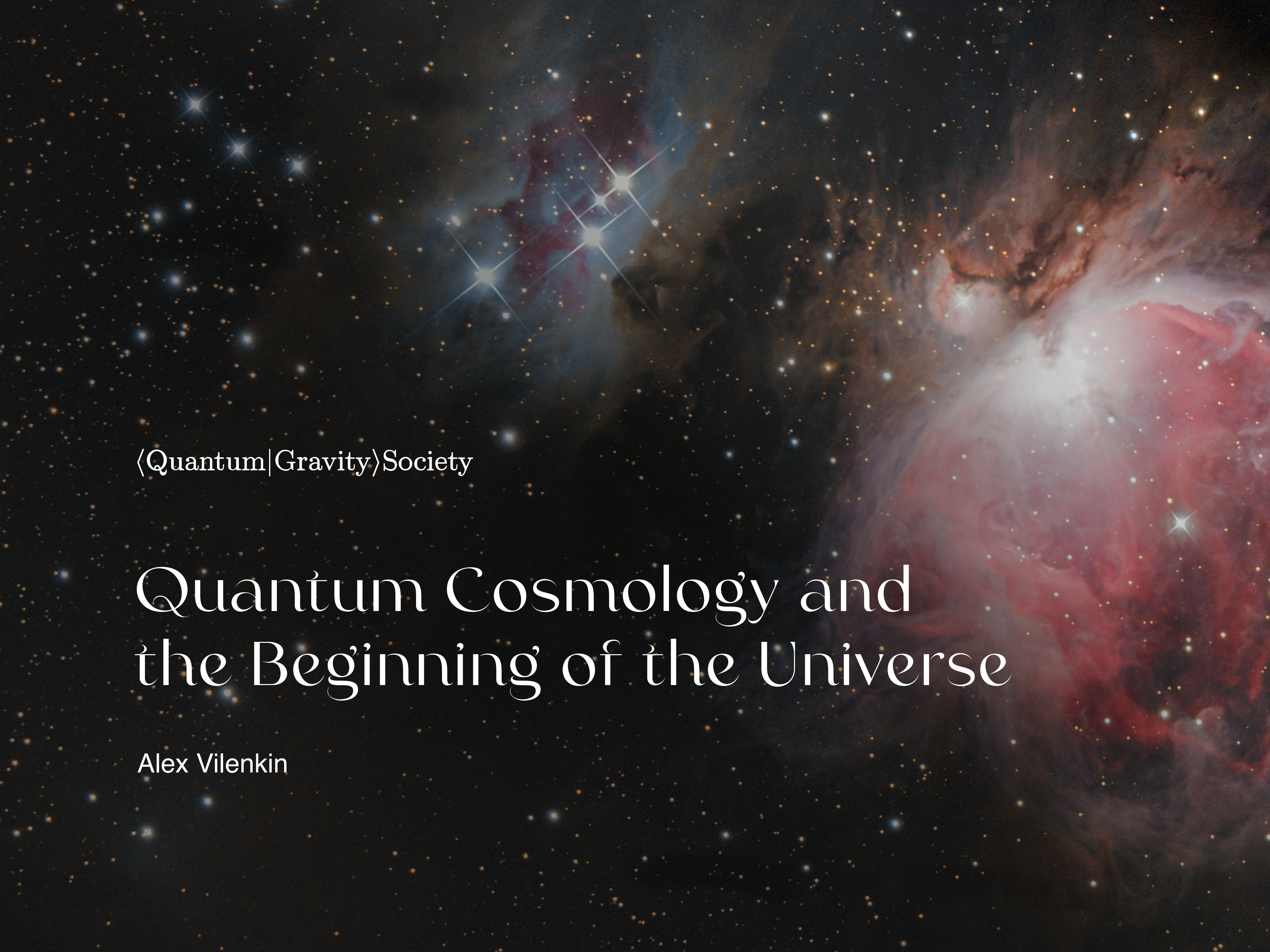

⟨Quantum|Gravity⟩Society

Quantum Cosmology and the Beginning of the Universe

Alex Vilenkin

QUANTUM COSMOLOGY AND THE BEGINNING OF THE UNIVERSE

Alex Vilenkin

Tufts Institute of Cosmology

Vancouver, August 2022

Did the universe have a beginning?

If so, what determined its initial state?

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This talk:

- *There probably was a beginning*
- *The initial state may be found from Quantum Cosmology*
- *Some conceptual problems of Quantum Cosmology*

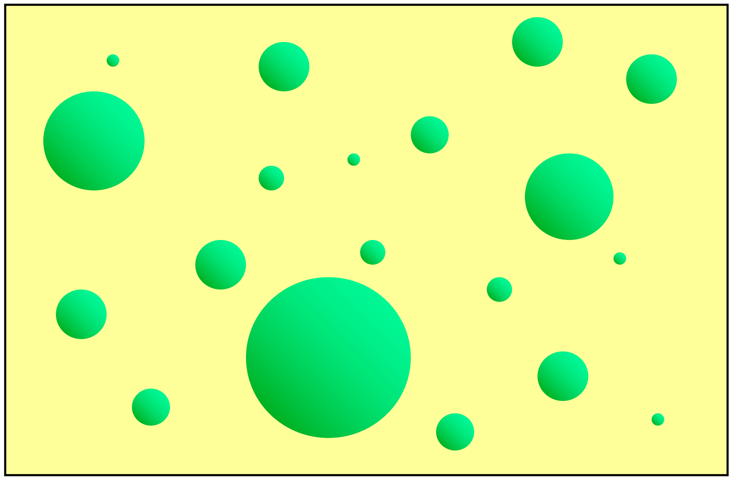
An eternal universe?

"Eternal inflation"

Cosmic inflation – very fast, accelerated expansion of the universe.

Explains some puzzling features of the big bang;
Predictions supported by observations.

Inflation is generically eternal to the future.



Bubbles nucleate and expand in the
Inflating background.

We live in one of the bubbles.

Could inflation have no beginning in the past?

BGV theorem:

A universe that is on average expanding is geodesically incomplete to the past.

Borde, Guth & A.V. (2003)

BGV theorem:

A universe that is on average expanding is geodesically incomplete to the past.

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Geodesics cannot be extended indefinitely to the past, except for a set of measure zero.

Does not assume any spacetime symmetries.
Does not rely on Einstein's equations.

⇨ *Inflationary spacetimes must have a past boundary (a beginning).*

Cyclic universe

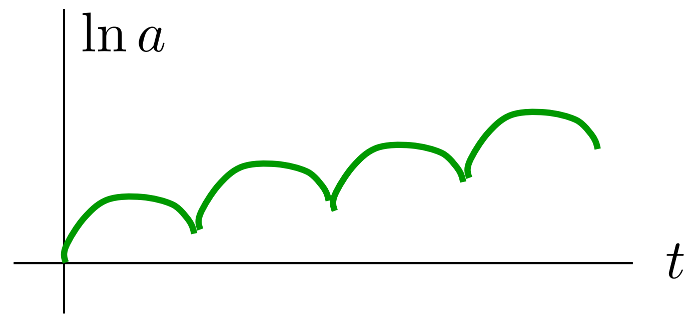
Steinhardt & Turok (2001)
Ijjas & Steinhardt (2019)

The problem with periodic cycles: entropy *S* will increase in every cycle.

⇨ Thermal death.

Tolman (1934):

The volume should grow in every cycle.
S grows, while *S/V* is bounded.



But in this model the universe is on average expanding
⇨ the spacetime is past-incomplete.

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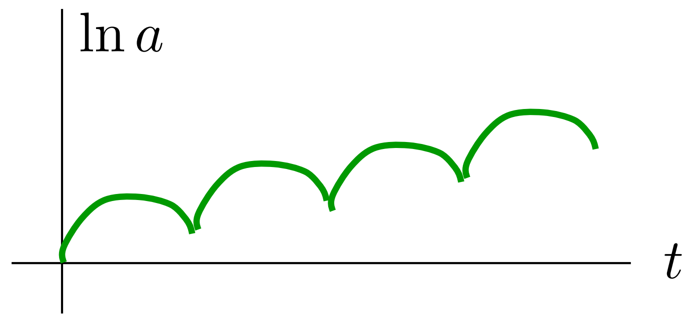

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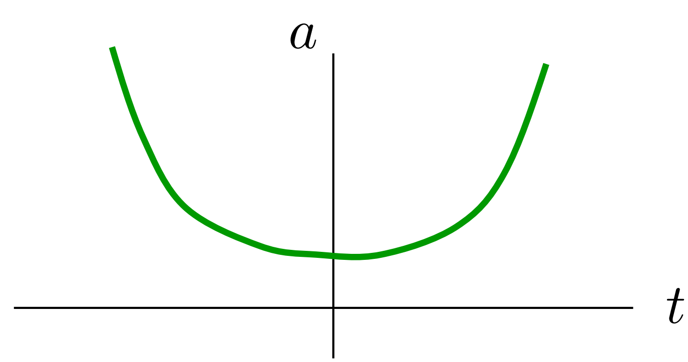
A bouncing universe

The universe contracts from infinite size.

No problem with BGV.



An inflationary universe must have a beginning.

Then what determined the initial conditions?

Quantum cosmology:

A compact inflating universe can spontaneously nucleate out of "nothing".

A state with no classical space and time.

A.V. (1982), Hartle & Hawking (1983), Linde (1984), Rubakov (1984), Zel'dovich & Starobinsky (1984)

Quantum cosmology

DeWitt (1967)

$\psi(g,\phi)$ – wave function of the universe

$\mathcal{H}\,\psi = 0$ – Wheeler-DeWitt equation

$$\left(\nabla^2 - U\right)\psi = 0$$

$\{g,\phi\}$ – "superspace"

3-geometry Matter fields

∇^2 Superspace Laplacian

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- ψ does not depend on time.

  A clock is a part of the universe. ⇒ Time should be defined in terms of $\{g,\phi\}$

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- How do we calculate probabilities? .

- What are the boundary conditions for ψ?

Defining probabilities

Should reduce to classical GR and ordinary QM in appropriate limits.

Conserved current:

$$J = i\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right), \qquad \nabla \cdot J = 0.$$

We can define probability on a hypersurface Σ in superspace:

$$dP = J \cdot d\Sigma$$

This guarantees unitarity:

$$\int J \cdot d\Sigma = 1$$

But *dP* is not positive definite.

Semiclassical approach

DeWitt (1967), Misner (1972)
Lapchinsky & Rubakov (1979)
A.V. (1989), Hartle (1993)

$$\{g,\phi\}\rightarrow\{c,q\}$$

Classical Quantum (a small subsystem)

Some semiclassical variables are necessary to define time.

$$\psi(c,q)=\underbrace{A(c)e^{iS(c)}}_{\text{Semiclassical}}\,\chi(c,q)$$

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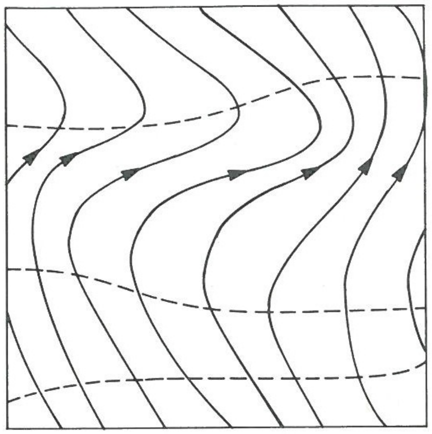
Leading order: $\nabla S \cdot \nabla S + U = 0$

Hamilton-Jacobi equation

The action *S(c)* describes an ensemble of classical universes : $\dot{c} = \nabla S$

$$J_c = |A^2| \nabla S \;, \qquad dP_c = J_c \cdot d\Sigma$$

The classical probabilities are positive if all classical trajectories cross Σ in the same direction.



Surfaces Σ play the role of equal time surfaces.

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$$\{g,\phi\} \to \{c,q\}$$

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Next order:

$$i\frac{\partial \chi}{\partial t} = H_q \chi$$

with time *t* defined by the classical ensemble.

$$dP = J \cdot d\Sigma = dP_c \left|\chi^2\right| dq$$

Recover the usual QM for the quantum subsystem.

Probabilities are defined only with the accuracy of the semiclassical approximation.

Boundary conditions

In ordinary QM the boundary conditions for ψ are determined by the physical setup external to the system.

But there is nothing external to the universe. $\Rightarrow$ The b.c. for ψ should be postulated as an independent physical law.

Path integral representation:

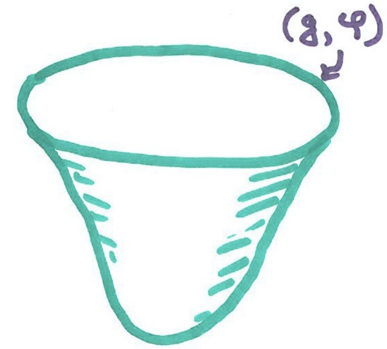
$$\psi(g,\phi) = \int^{(g,\phi)} \mathcal{D}g\mathcal{D}\phi e^{iS}$$

What is the class of paths?

Hartle-Hawking wave function

Hartle & Hawking (1983)
Halliwell, Hartle & Hertog (2019)

$$\psi_{HH}(g,\phi) = \int^{(g,\phi)} e^{-S_E}$$



Euclidean path integral: $t \rightarrow i\tau$.

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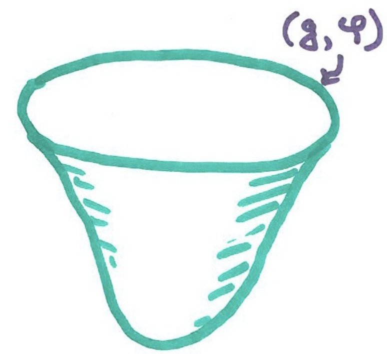

Euclidean path integral: $t \to i\tau$.

A problem:

The Euclidean action is unbounded from below, so the path integral is divergent.

HH: Integrate over complex metrics. But how do we choose the contour?

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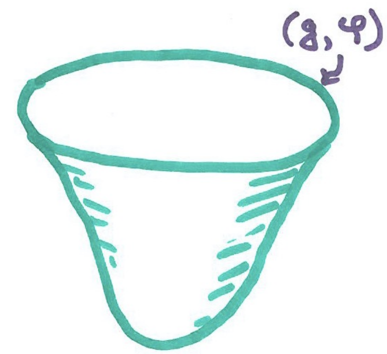

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Application to inflationary cosmology

A spherical universe of radius a and a scalar field ϕ with a potential $V(\phi)$.

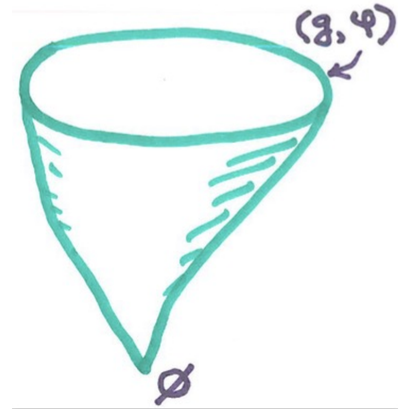
Use a as time and find probability distribution for the initial values of ϕ :

$$P(\phi) \sim \exp\left(\frac{3}{8V(\phi)}\right) d\phi$$

The most probable initial state has the lowest energy density and the largest radius. Disfavors inflation.

Tunneling wave function

A.V. (1984)
A.V. & Yamada (2018)



$$\psi_T(g,\phi) = \int_{\varnothing}^{(g,\phi)} e^{iS}$$

Lorentzian path integral.

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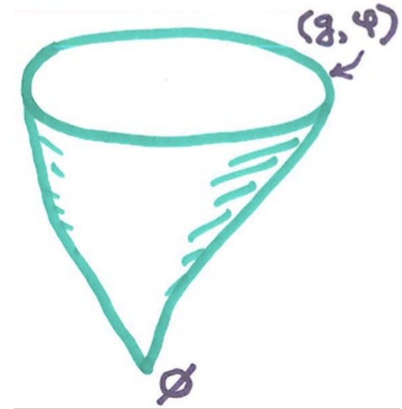


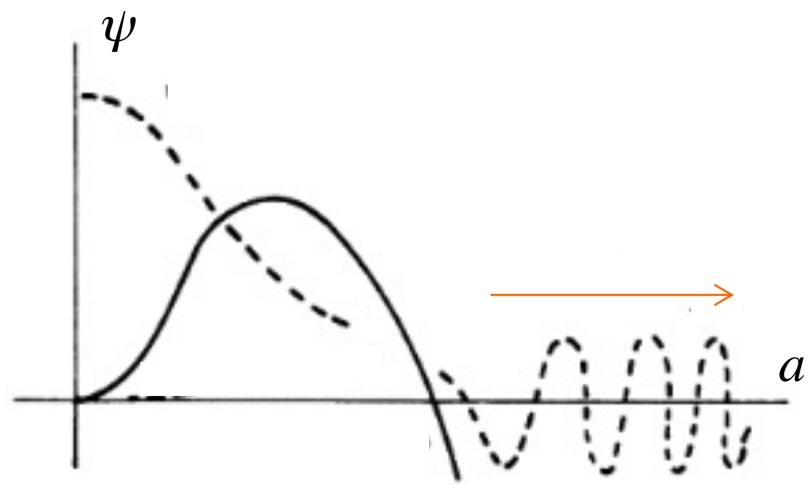
$$\psi_T(g,\phi) = \int_{\varnothing}^{(g,\phi)} e^{iS}$$

Lorentzian path integral.

For a spherical universe:

The wave function has the form suggestive of quantum tunneling from zero size to a finite radius.

$$dP(\phi) \ \ \exp\left(-\frac{3}{8V(\phi)}\right) d\phi$$



The most probable initial state has the highest vacuum energy density and smallest initial radius. Favorable for inflation.

Conclusions

- Inflationary cosmology is incomplete without initial conditions for the universe.

- The wave function of the universe may provide the probability distribution for the initial states. But we need b.c. for ψ.

- The "tunneling" wave function favors initial states suitable for inflation.

- In quantum cosmology, *probability* and *unitarity* are approximate concepts. They are defined only with the accuracy of the semiclassical approximation. *Is this a satisfactory situation?*

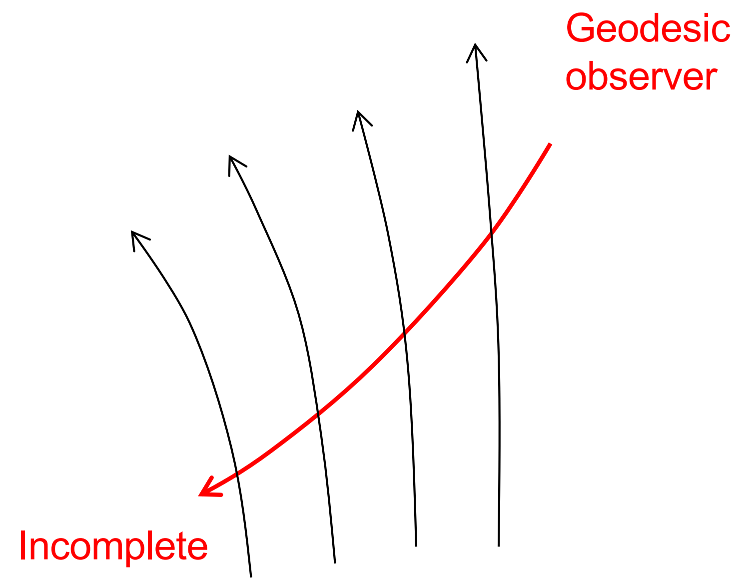
- Can boundary conditions for ψ be derived from a fundamental theory?

A more precise statement of the theorem:

Imagine the spacetime is filled with a congruence of test particles.

Let $H(\tau)$ be the Hubble parameterof the congruence along the worldline of some geodesic observer.

If $H_{av} > 0$ along the worldline, this worldline must be past-incomplete.



Congruence of test particles

Worldlines cannot be extended indefinitely to the past, except perhaps for a set of measure zero.

⇨ *Inflationary spacetimes must have a past boundary (a beginning).*

Does not rely on Einstein's equations or any conditions like NCC.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