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Unruh, Cherenkov and Hawking Radiation

From a negative frequency perspective and generation of entangled photon pairs

Anatoly Svidzinsky, together with Marlan Scully and Bill Unruh

Unruh, Cherenkov and Hawking radiation from a negative frequency perspective and generation of entangled photon pairs

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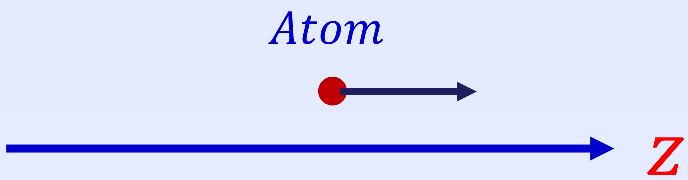
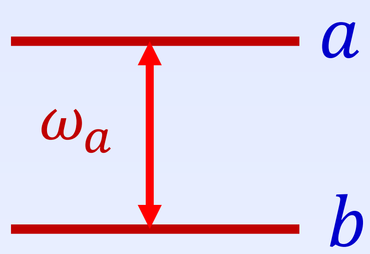
Texas A&M University, USA

Interaction between atoms and the field

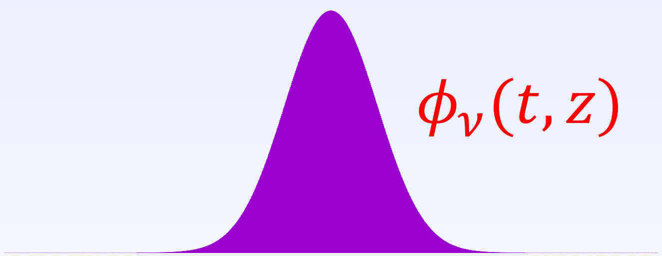




Atom's trajectory

$$t = t(\tau)$$

$$z = z(\tau)$$



τ is the proper time of atom

$\hat{a}_\nu^+$ is photon creation operator

Interaction Hamiltonian between the atom and a photon with mode function $\phi_\nu(t, z)$ is

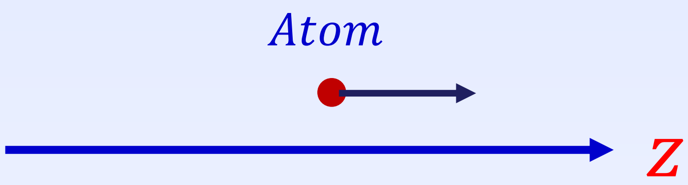
$$\hat{V}(\tau) = \hbar g[\phi_\nu(t(\tau), z(\tau))\hat{a}_\nu + \phi_\nu^*(t(\tau), z(\tau))\hat{a}_\nu^+]\left(\hat{\sigma} e^{-i\omega_a \tau} + \hat{\sigma}^+ e^{i\omega_a \tau}\right)$$

where the field mode function is taken at the atom's location

$\hat{a}_\nu$ is photon annihilation operator, $\hat{\sigma}$ is atom's lowering operator

Atom feels the local value of the field at the atom's location.
Local properties of the photon mode function determine the atom's ability to emit and absorb the photon.

If along atom's trajectory

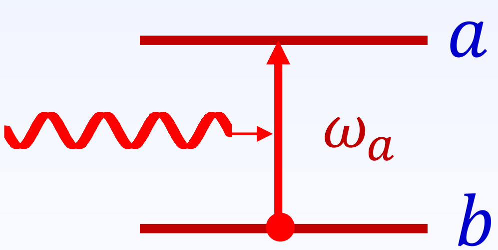


$$\phi_\nu(t(\tau), z(\tau)) \propto e^{-i\nu\tau}$$

then atom feels that field harmonically oscillates with time at frequency ν

If

$$\nu = \omega_a$$



then the term in the interaction Hamiltonian

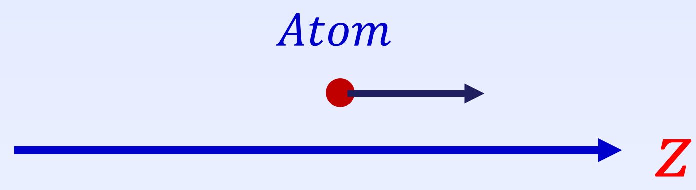
$$\hat{\sigma}^+ \hat{a}_\nu e^{i\omega_a \tau} \phi_\nu(t(\tau), z(\tau)) \propto \hat{\sigma}^+ \hat{a}_\nu e^{i(\omega_a - \nu)\tau}$$

yields resonant excitation of the atom with photon absorption

From the atom's perspective, the photon has positive frequency

Atom feels the local value of the field at the atom's location.
Local properties of the photon mode function determine the atom's ability to emit and absorb the photon.

If along atom's trajectory

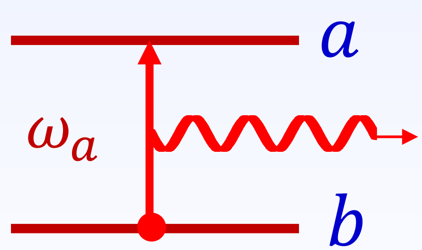


$$\phi_\nu(t(\tau), z(\tau)) \propto e^{-i\nu\tau}$$

then atom feels that field harmonically oscillates with time at frequency ν

If

$$\nu = -\omega_a$$



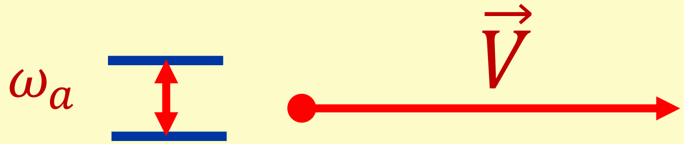
then the term in the interaction Hamiltonian

$$\hat{\sigma}^+ \hat{a}_\nu^+ e^{i\omega_a\tau} \phi_\nu^*(t(\tau), z(\tau)) \propto \hat{\sigma}^+ \hat{a}_\nu^+ e^{i(\nu+\omega_a)\tau}$$

yields resonant excitation of the atom with photon emission

From the atom's perspective, the emitted photon has negative frequency

Atom moving through a medium



Refractive index n

Atom's trajectory

$$t = \frac{\tau}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad \vec{r} = \frac{\vec{V}\tau}{\sqrt{1 - \frac{V^2}{c^2}}}$$

τ is the proper time of atom

Photon mode functions

$$\phi_k(t, \vec{r}) = e^{-i\frac{ck}{n}t + i\vec{k}\cdot\vec{r}}$$

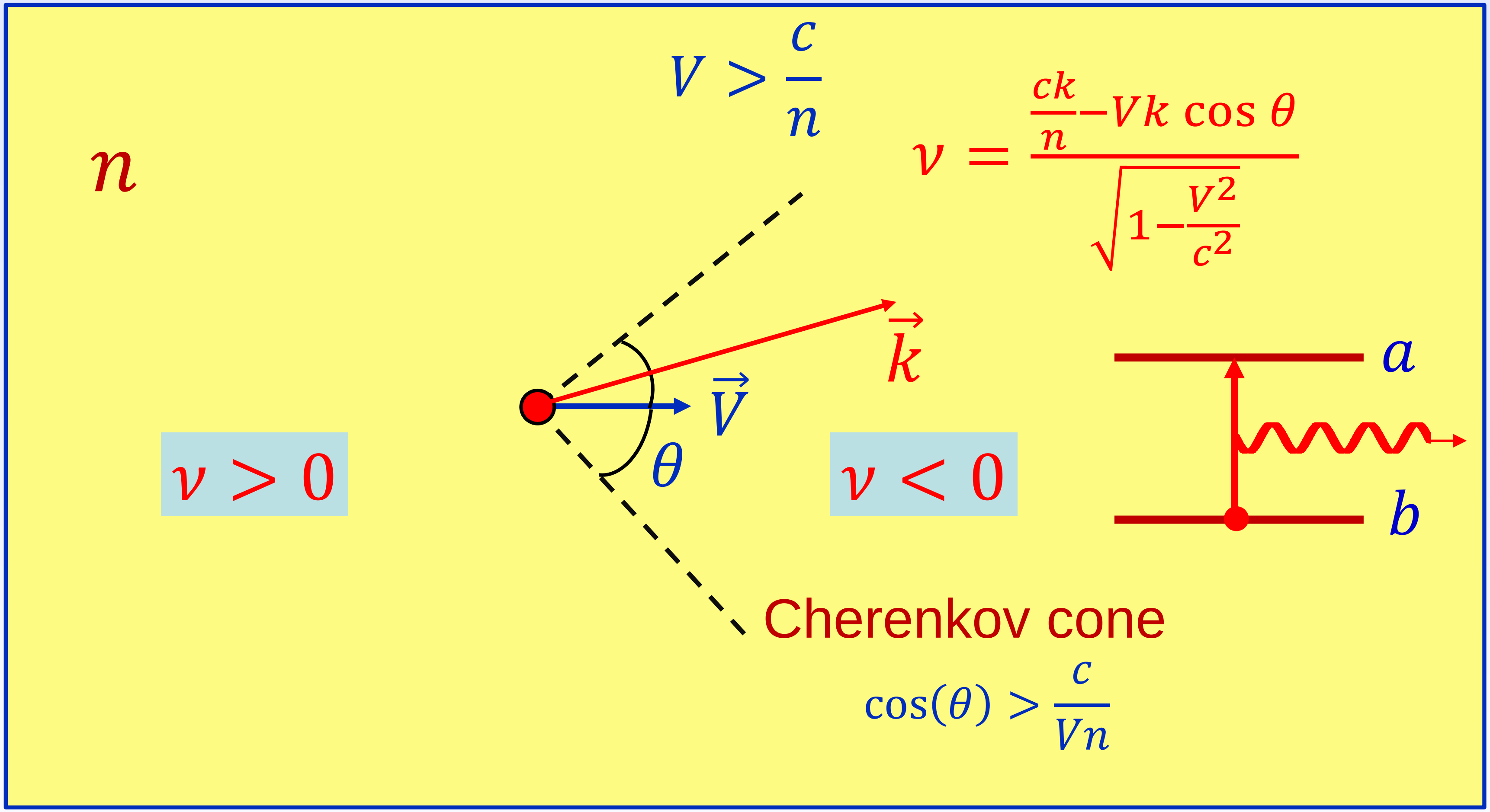
Along the atom's trajectory the mode function is

$$\phi_k(t(\tau), \vec{r}(\tau)) = e^{-i\nu\tau}$$

where

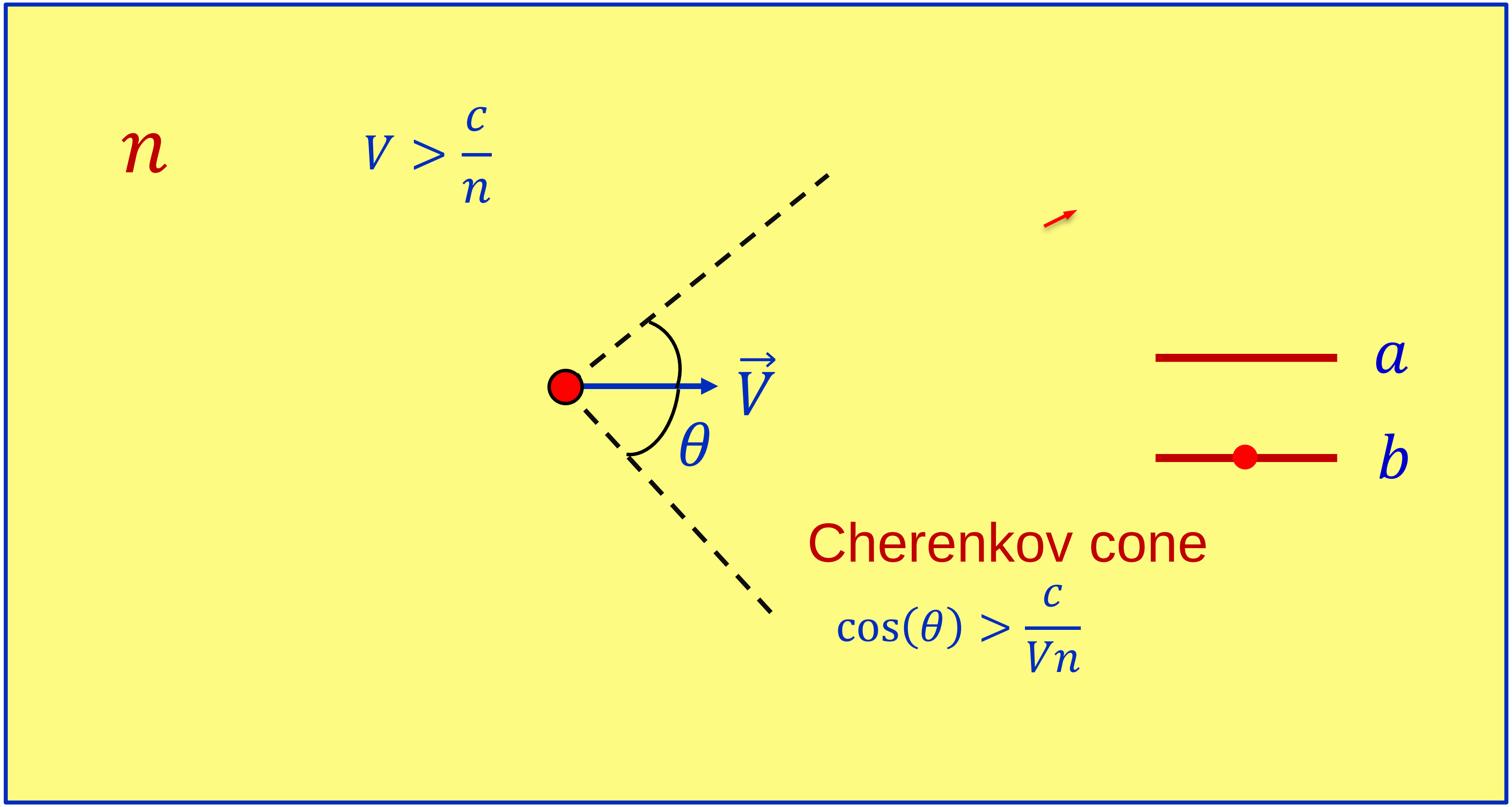
$$\nu = \frac{\frac{ck}{n} - \vec{V}\cdot\vec{k}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

If $V > \frac{c}{n}$ the photon frequency can be negative from atom's perspective

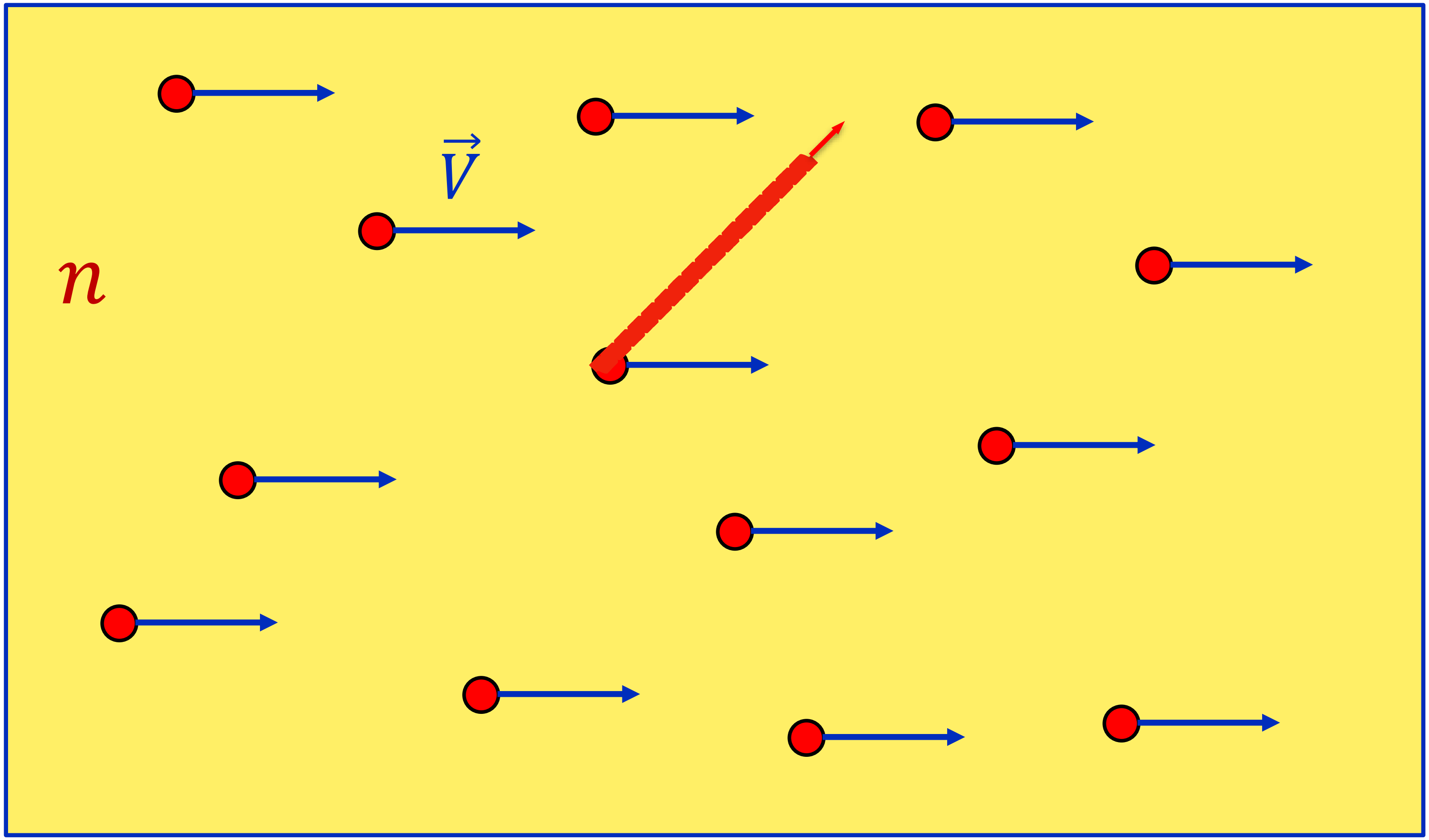


From atom's perspective, photon propagating in the direction inside (outside) the Cherenkov cone has negative (positive) frequency.

Moving atom can emit photon into Cherenkov cone and become excited (Cherenkov radiation)



Photon propagating in the direction inside the Cherenkov cone can not be absorbed by the ground-state atom.

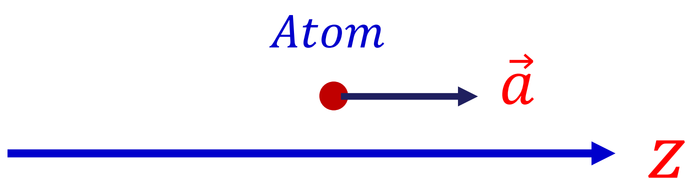


Ensemble of atoms moving through a medium with equal velocity $V > c/n$ and emitting Cherenkov radiation.

In the moving frame, the Cherenkov photons emitted by the atoms have negative frequency. As a result, emitted photons can not be absorbed by other ground-state atoms in the ensemble ("inverted" medium).

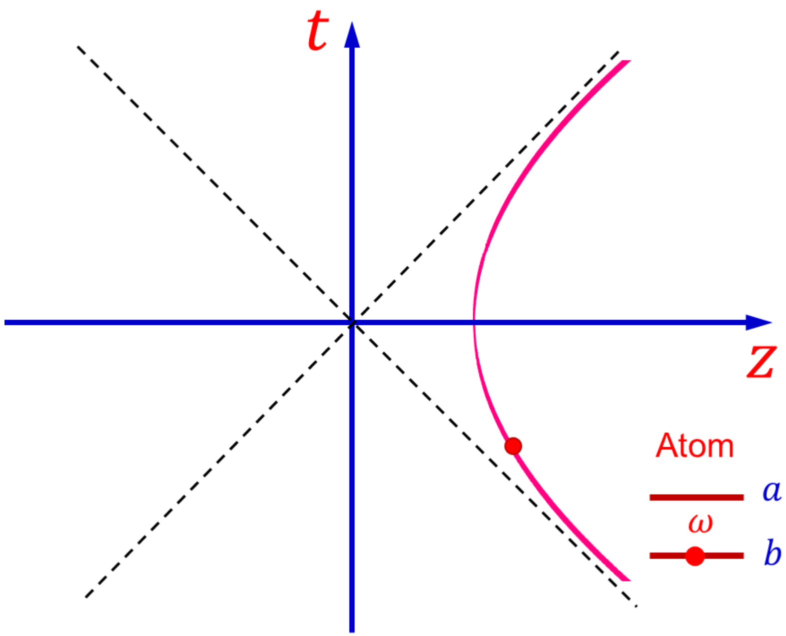
Uniformly accelerated atom in Minkowski vacuum

The atom trajectory is



$$t = \frac{c}{a} sinh\left(\frac{a\tau}{c}\right)$$

$$z = \frac{c^2}{a} cosh\left(\frac{a\tau}{c}\right)$$



τ is the proper time of atom

Unruh-Minkowski modes $(\Omega > 0)$

$$F_{1\Omega}(t,z) = \frac{|t \pm z/c|^{i\Omega}}{\sqrt{2\Omega\ \sinh(\pi\Omega)}} \begin{cases} e^{-\pi\Omega/2}, & t \pm \frac{z}{c} > 0 \\ e^{\pi\Omega/2}, & t \pm \frac{z}{c} < 0 \end{cases}$$

$$F_{2\Omega}(t,z) = \frac{|t \pm z/c|^{-i\Omega}}{\sqrt{2\Omega\ \sinh(\pi\Omega)}} \begin{cases} e^{\pi\Omega/2}, & t \pm \frac{z}{c} > 0 \\ e^{-\pi\Omega/2}, & t \pm \frac{z}{c} < 0 \end{cases}$$

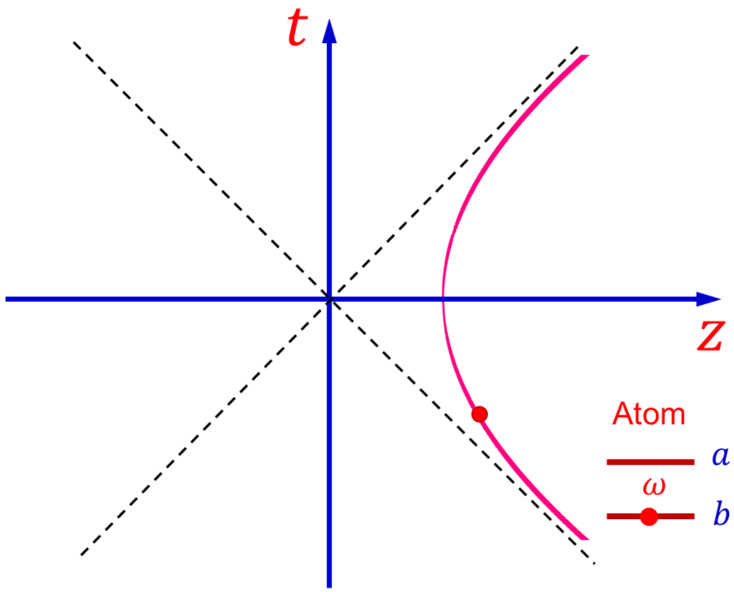
where $\Omega > 0$ is a parameter

$$F_{2\Omega} = F_{1(-\Omega)}$$

The Unruh-Minkowski modes form a complete set and have positive norm.

In Minkowski vacuum there are no Unruh-Minkowski photons

$$\langle 0_M | \hat{a}^+_\Omega \hat{a}_\Omega | 0_M \rangle = 0$$



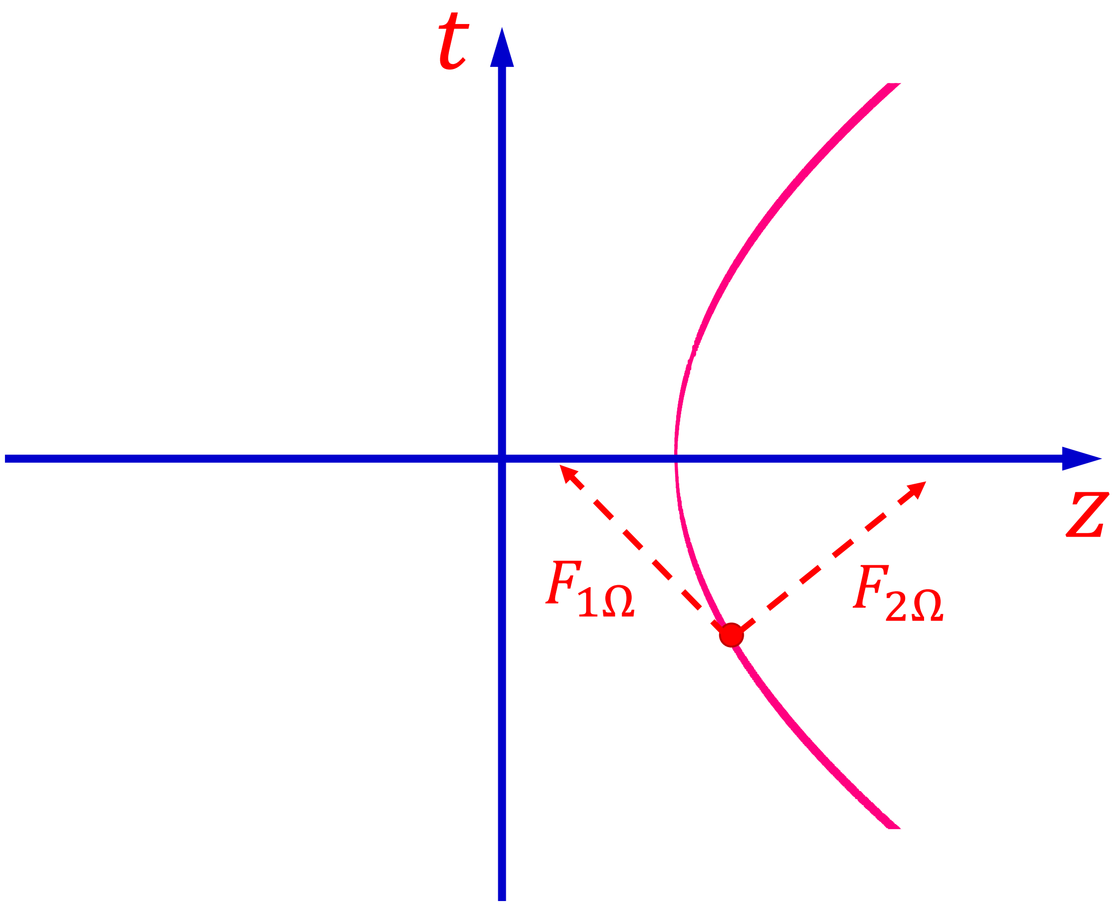
Along the worldline of the accelerated atom the Unruh-Minkowski mode functions for the left and right propagating photons are

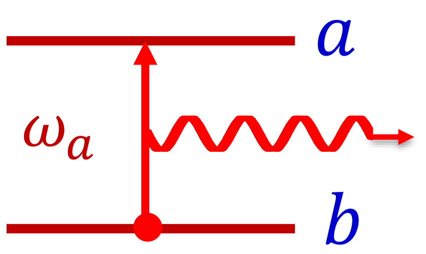
$$F_{R1\Omega}, F_{L2\Omega} \propto e^{-ia\Omega\tau/c} \qquad \nu = \frac{a\Omega}{c}$$

$$F_{L1\Omega}, F_{R2\Omega} \propto e^{ia\Omega\tau/c} \qquad \nu = -\frac{a\Omega}{c}$$

From the perspective of the atom accelerated in the right Rindler wedge the mode functions $F_{L1\Omega}$ and $F_{R2\Omega}$ have negative frequency

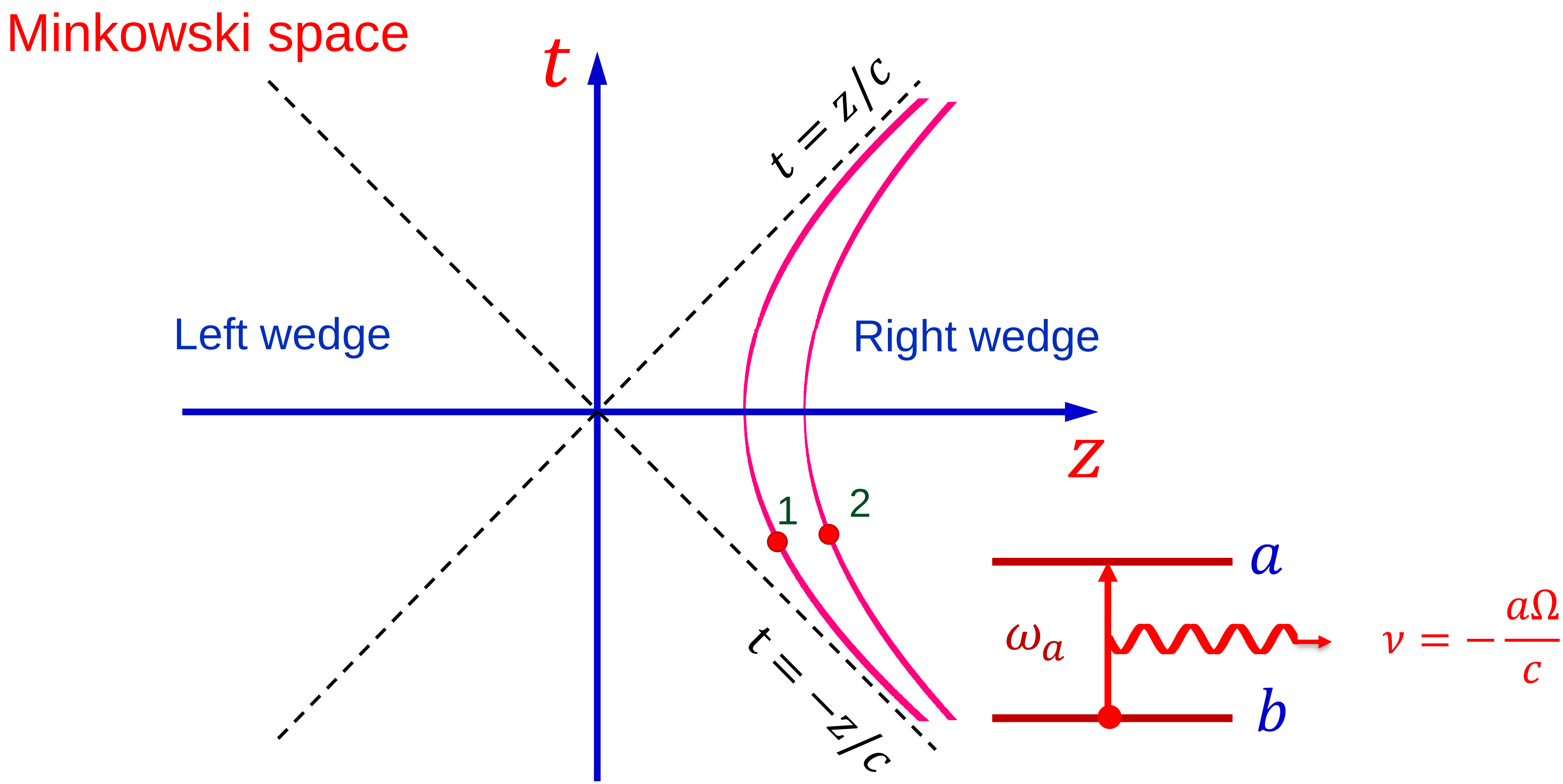
Unruh acceleration radiation



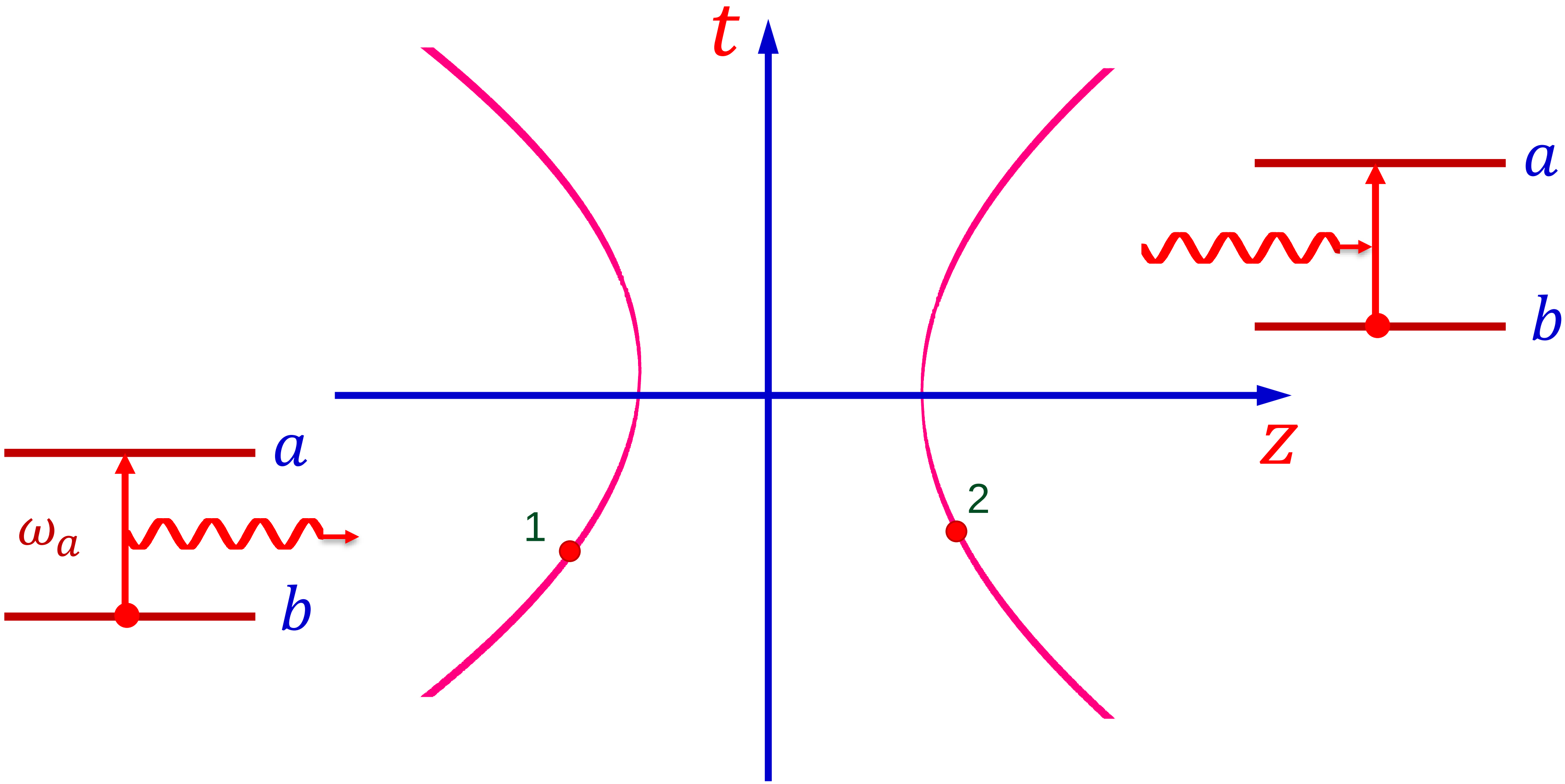


A ground-state atom moving in the right Rindler wedge with acceleration a can emit the left-propagating photon into the Unruh-Minkowski mode $F_{1\Omega}$ and the right-propagating photon into the mode $F_{2\Omega}$

$$\Omega = \frac{c\omega_a}{a}$$



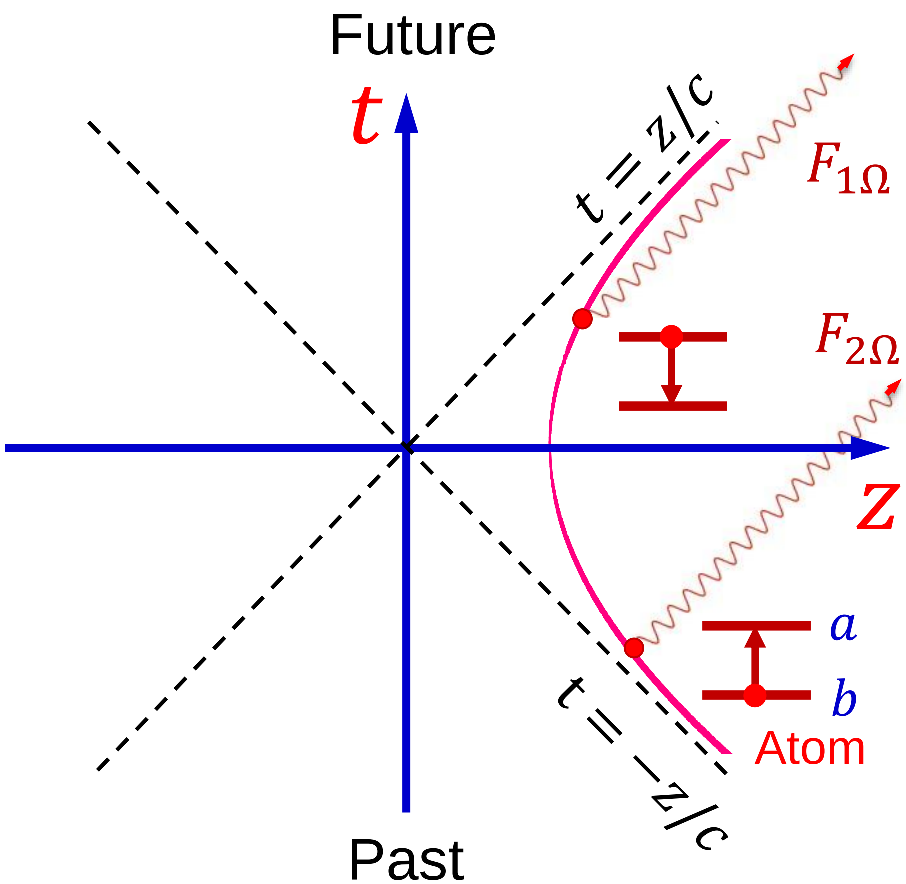
Ground-state atom 2 accelerated in the same direction can not become excited by absorbing a photon emitted by atom 1 because the emitted photon has negative frequency from the perspective of atoms accelerated in the same direction ($P_{a_1 a_2 0} = 0$).



If atoms are accelerated in opposite directions, then from atom’s perspective the normal mode frequencies have opposite sign.

Thus, Unruh-Minkowski photon emitted by atom 1 can be absorbed by the ground-state atom 2.

Generation of entangled photon pairs by accelerated atom



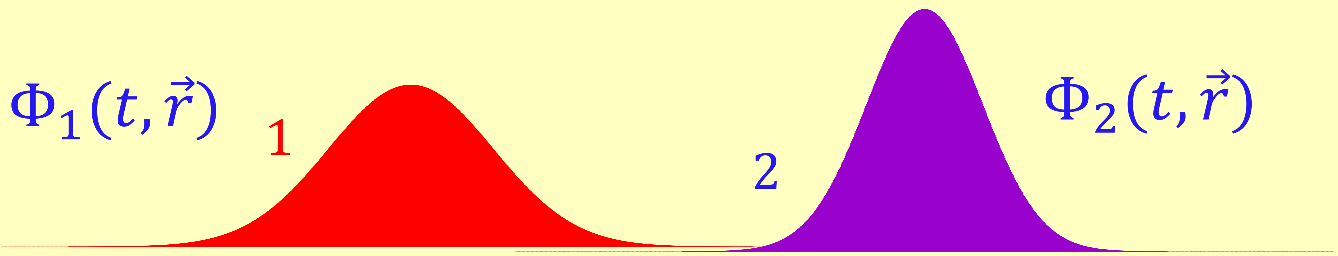
Final state of the field

$$|\Psi\rangle \approx (1 + G\hat{a}^{+}_{2\Omega}\hat{a}^{+}_{1\Omega})|0_M\rangle$$

Two-mode squeezed state

Atom emits entangled pairs of Unruh-Minkowski photons

Two-mode squeezed state (photon number correlation)



$$|\psi\rangle = e^{\gamma^* \hat{a}_1^+ \hat{a}_2^+ + \gamma \hat{a}_1 \hat{a}_2} |0_1 0_2\rangle$$ ⟵ State is entangled

Tracing over one of the modes leaves the remaining mode in a thermal state

Fluctuations of the particle number (variance)

$$\Delta n_1 = \sqrt{\langle n_1^2\rangle - \langle n_1\rangle^2} = \sqrt{\bar{n}(1+\bar{n})}$$ ⟵ The same as for thermal state

$$\Delta n_1 = \Delta n_2$$

where $\bar{n}$ is the average number of photons in each mode

$$\Delta(n_1 - n_2) = 0$$ ⟵ Photon numbers in modes are correlated

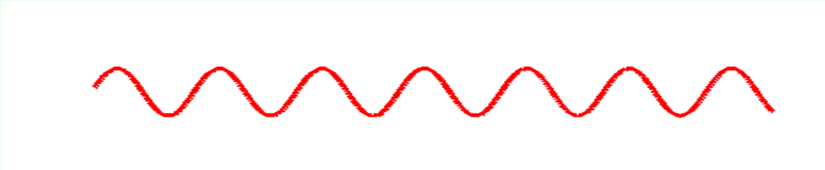
If there are n photons in mode 1 then with unit probability there are n photons in mode 2

Particle content of a state depends on mode functions we choose to quantize the field

Minkowski vacuum

Plane-wave modes

$$\phi_\nu(t,z) = \frac{1}{\sqrt{2\nu}} e^{-i\nu\left(t \pm \frac{z}{c}\right)}$$



Or Unruh-Minkowski modes

Rindler modes

$$\phi_{1\nu} = e^{i\frac{\nu c}{a} ln(\mp z - ct)} \theta(\mp z - ct)$$

$$\phi_{2\nu} = e^{-i\frac{\nu c}{a} ln(ct \pm z)} \theta(ct \pm z)$$

where a is a parameter which has dimension of acceleration

Average number of photons in the mode ν

$$\langle 0_M | \hat{a}_\nu^+ \hat{a}_\nu | 0_M \rangle = 0$$

There are no plane-wave photons or Unruh-Minkowski photons in Minkowski vacuum

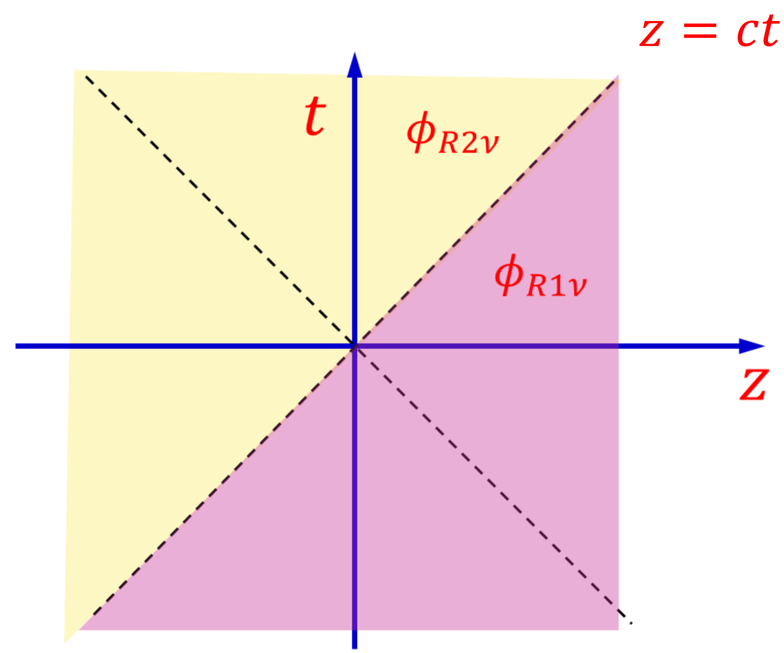
$$\langle 0_M | \hat{b}_\nu^+ \hat{b}_\nu | 0_M \rangle = \frac{1}{exp\left(\frac{\hbar\nu}{k_B T_U}\right) - 1}$$

where $T_U = \frac{\hbar a}{2\pi c k_B}$ is Unruh temperature

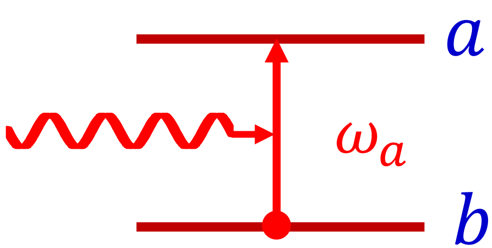
State is filled with Rindler photons

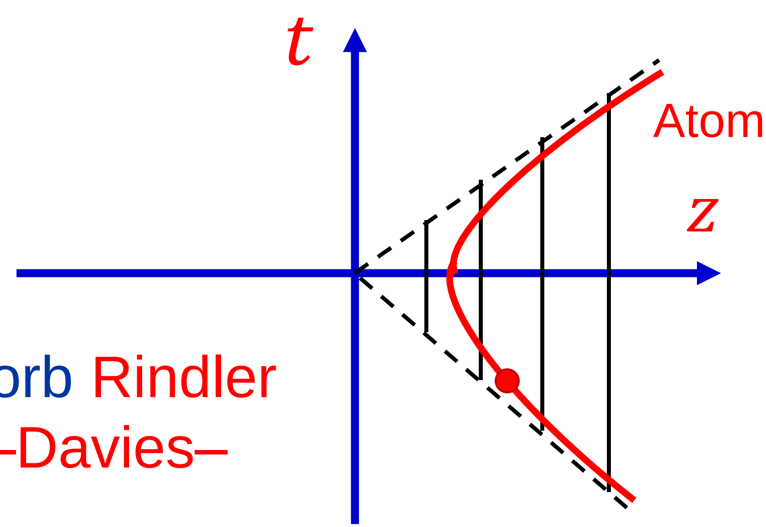
Numbers of Rindler photons in causally disconnected regions are correlated in Minkowski vacuum.

If there is Rindler photon in region 1 then with unit probability there is photon in region 2



Minkowski vacuum is entangled (squeezed state)



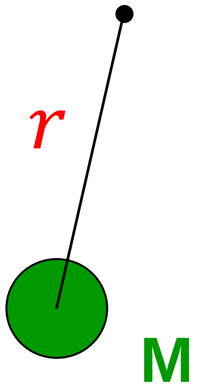


Uniformly accelerated atoms can absorb Rindler photons and become excited (Fulling–Davies–Unruh effect)

Insights on Hawking radiation

Gravitational field of hypothetical static black hole in 1+1 dimension is described by the Schwarzschild metric

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{1}{1 - \frac{r_g}{r}} dr^2$$



where $r_g = \frac{2GM}{c^2}$ is the gravitational radius

Kruskal-Szekeres coordinates

$r > r_g$

$$T = \sqrt{\frac{r}{r_g} - 1} e^{\frac{r}{2r_g}} \sinh\left(\frac{ct}{2r_g}\right)$$

$$X = \sqrt{\frac{r}{r_g} - 1} e^{\frac{r}{2r_g}} \cosh\left(\frac{ct}{2r_g}\right)$$

$r < r_g$

$$T = \sqrt{1 - \frac{r}{r_g}} e^{\frac{r}{2r_g}} \cosh\left(\frac{ct}{2r_g}\right)$$

$$X = \sqrt{1 - \frac{r}{r_g}} e^{\frac{r}{2r_g}} \sinh\left(\frac{ct}{2r_g}\right)$$

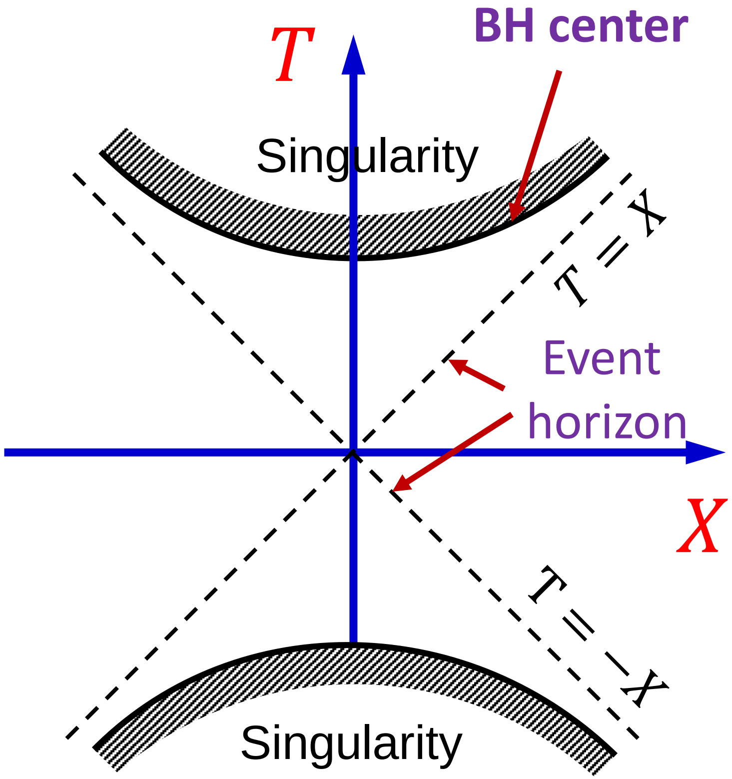
Metric is conformal:

$$ds^2 = \frac{4r_g^3}{r} e^{-r/r_g} (dT^2 - dX^2)$$

Massless scalar field ϕ obeys wave equation

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0$$

Maximally extended Schwarzschild spacetime

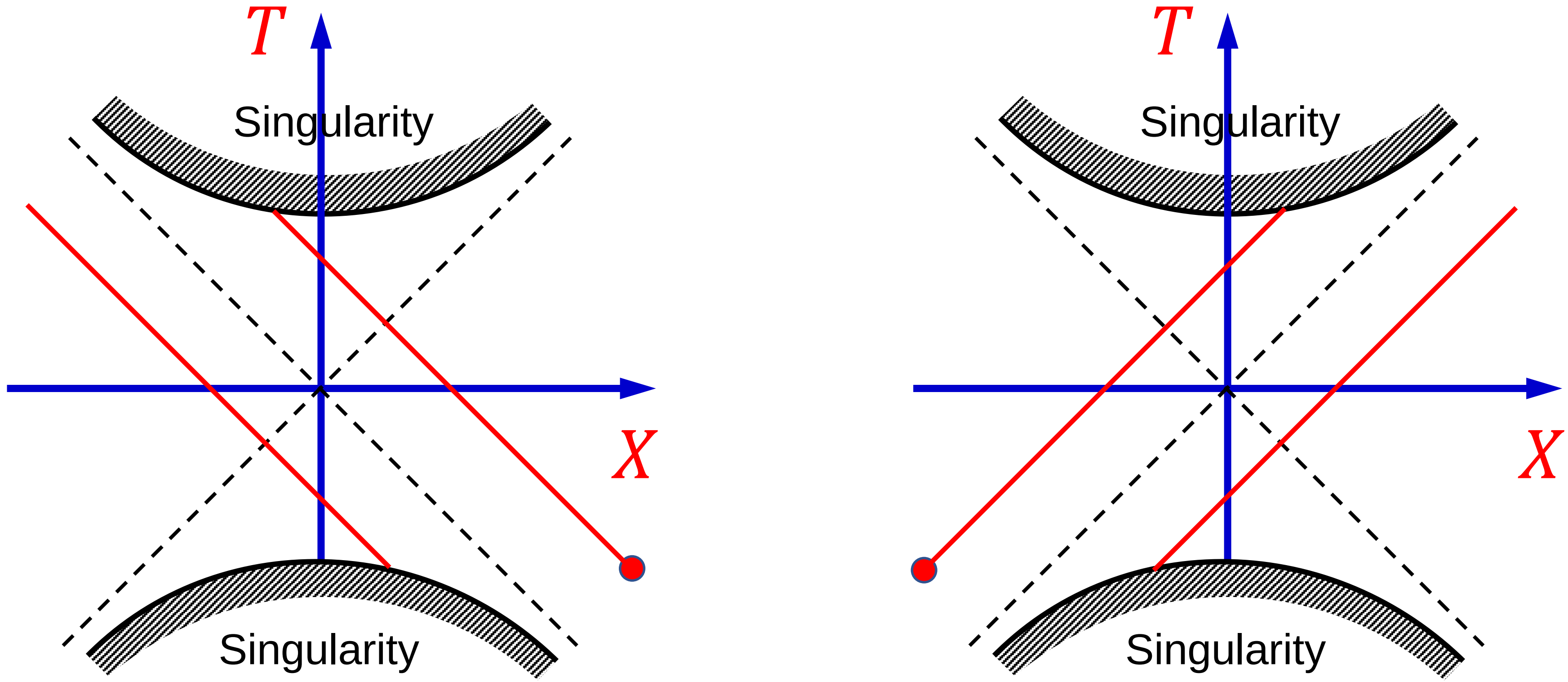


Unruh-Schwarzschild modes $(\Omega > 0)$

$$F_{1\Omega}(T,X) = \frac{|T \pm X|^{i\Omega}}{\sqrt{2\Omega\ \sinh(\pi\Omega)}} \begin{cases} e^{-\pi\Omega/2}, T \pm X > 0 \\ e^{\pi\Omega/2}, T \pm X < 0 \end{cases}$$

$$F_{2\Omega}(T,X) = \frac{|T \pm X|^{-i\Omega}}{\sqrt{2\Omega\ \sinh(\pi\Omega)}} \begin{cases} e^{\pi\Omega/2}, T \pm X > 0 \\ e^{-\pi\Omega/2}, T \pm X < 0 \end{cases}$$

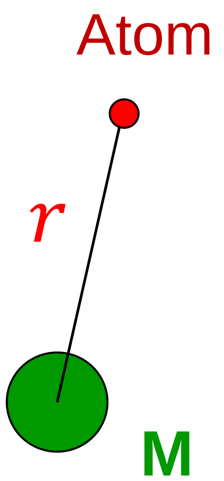
Maximally extended Schwarzschild spacetime

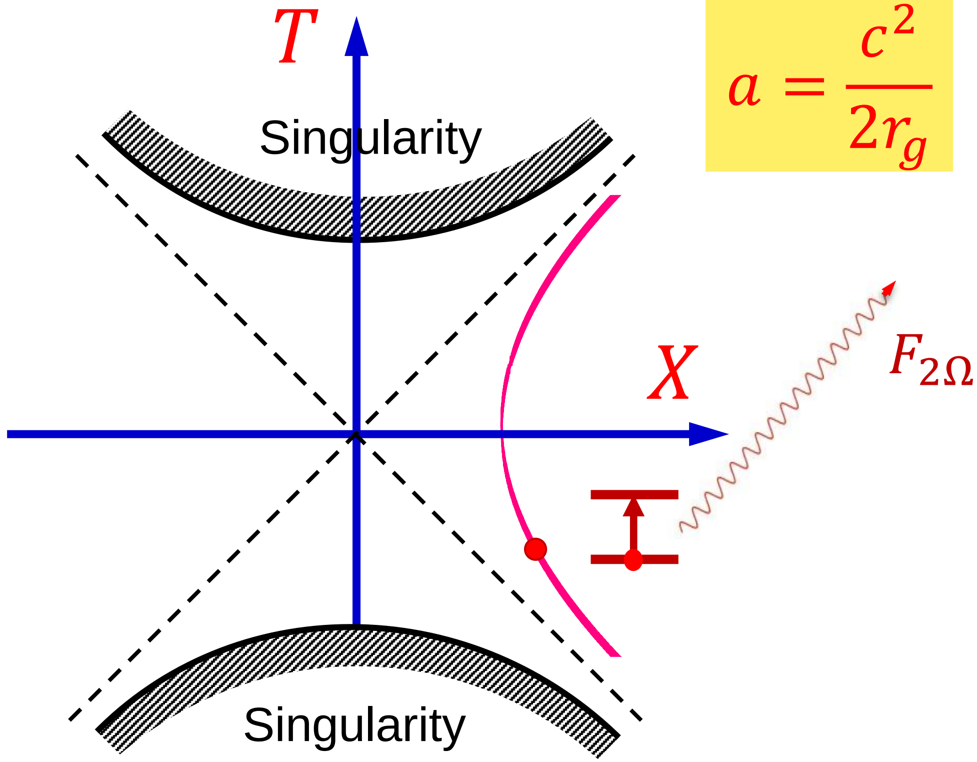


Modes disappear from the spacetime at the future singularity and appear at the past singularity

Assumption: field is in the vacuum state for Unruh-Schwarzschild photons, or equivalently, for plane waves $e^{-i\nu(T\pm X)}$ $(\nu > 0)$

Atom held fixed above horizon of a black hole ($r = const$) is uniformly accelerated in Kruskal-Szekeres coordinates

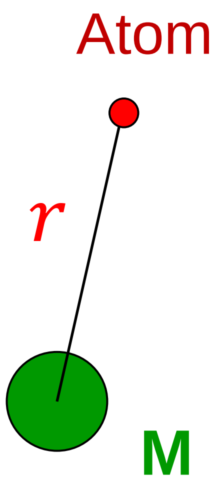


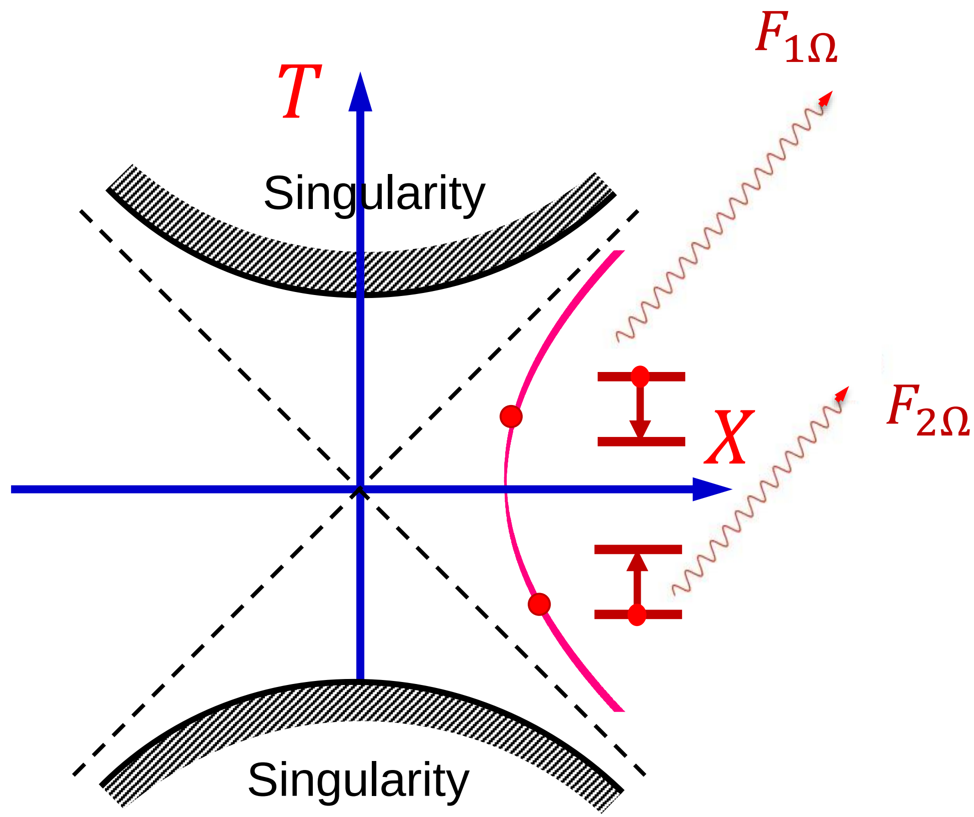


$$a = \frac{c^2}{2r_g}$$

Atom can become excited by emitting US photon, which can be interpreted as if atom detects radiation (Hawking radiation)

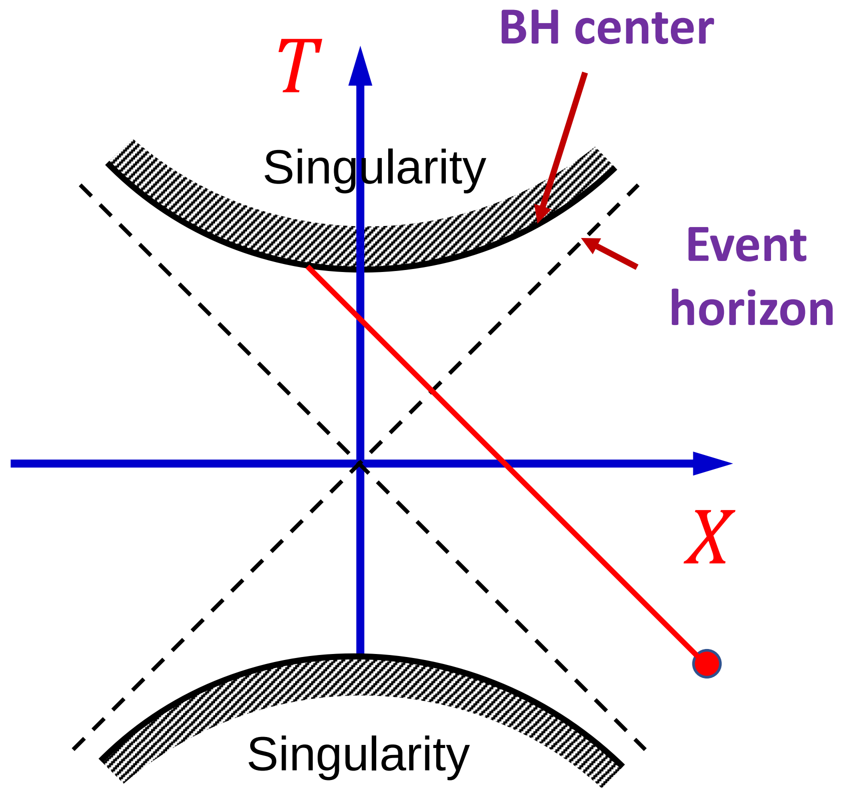
Unruh temperature: $T_U = \frac{\hbar a}{2\pi c k_B} = \frac{\hbar c^3}{8\pi G k_B M}$ ⟵ Hawking temperature





Excited atom can decay back to the ground state by emitting US photon into mode $F_{1\Omega}$. This yields generation of entangled pairs of Unruh-Schwarzschild photons in squeezed state:

$$|\Psi\rangle \approx (1 + G\hat{a}^{+}_{2\Omega}\hat{a}^{+}_{1\Omega})|0\rangle$$

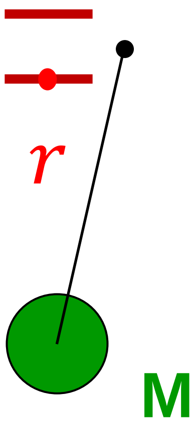


Hawking radiation appears because:

- BH center is perfectly absorbing spacetime boundary
- Field is in the vacuum state for $e^{-i\nu(T\pm X)}$ $(\nu > 0)$ modes

Different state of the field

Assume that field is in the vacuum state for photons described by mode functions (Boulware vacuum)



$$\phi_\nu(t,r) = e^{-i\nu\left(t\pm\frac{r}{c}\pm\ln\left|r-r_g\right|\right)}, \nu > 0$$

← Schwarzschild coordinates

Atom held fixed above BH horizon at $r = const$ will not become excited

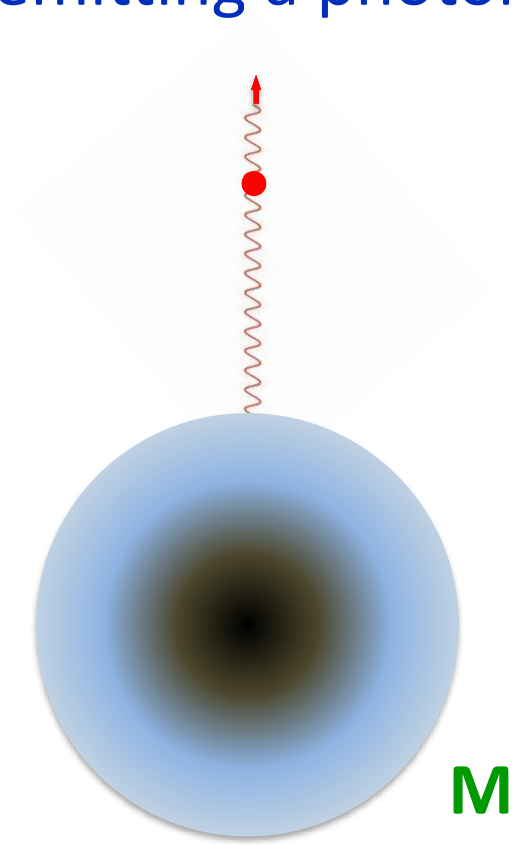
Free falling atom can become excited by emitting a photon

Excitation probability

$$P_\nu \propto \frac{1}{exp\left(\frac{4\pi r_g \nu}{c}\right) - 1}$$

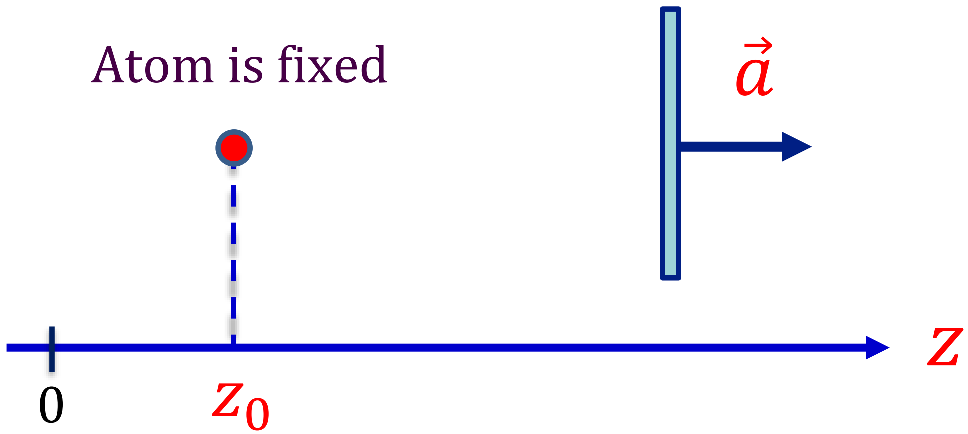
Planck factor with Hawking temperature. There is photon frequency ν under the exponential.

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

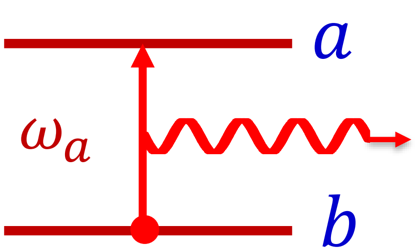


Photons are mainly emitted into outgoing modes

This is analogous to excitation of a fixed atom by uniformly accelerated mirror in Minkowski space-time



Probability that photon with frequency ν is emitted and atom is excited



$$P \propto \frac{1}{exp\left(\frac{2\pi\nu c}{a}\right) - 1}$$

← Planck factor with Unruh temperature

$$T_U = \frac{\hbar a}{2\pi c k_B}$$

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THE END


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